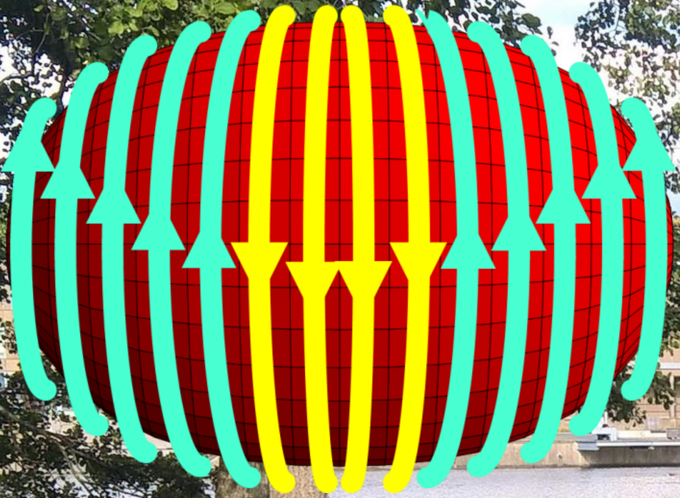

The magnetic octupole moment of ^{45}Sc

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University of York
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Outline

1. Basic definitions of nuclear multipole moments
2. Visualisation of the magnetic multipole moments in axial symmetry
3. Hartree-Fock + angular momentum projection
4. Deformation energies and electric quadrupole moment in ^{45}Sc
5. Magnetic dipole and octupole moments in ^{45}Sc
6. *Ab initio* magnetic moments in light nuclei
7. Conclusions

Basic definitions

The electric and magnetic moments are defined as

$$Q_{\lambda\mu} = \langle\Psi|\hat{Q}_{\lambda\mu}|\Psi\rangle = \int q_{\lambda\mu}(\vec{r})\, d^3\vec{r},$$

$$M_{\lambda\mu} = \langle\Psi|\hat{M}_{\lambda\mu}|\Psi\rangle = \int m_{\lambda\mu}(\vec{r})\, d^3\vec{r},$$

where $|\Psi\rangle$ is a many-body state, and $q_{\lambda\mu}(\vec{r})$ and $m_{\lambda\mu}(\vec{r})$ are the corresponding electric and magnetic-moment densities:

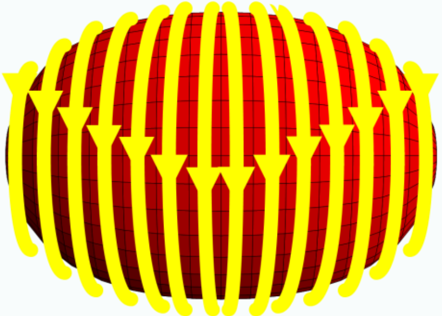
$$q_{\lambda\mu}(\vec{r}) = e\rho(\vec{r})Q_{\lambda\mu}(\vec{r}),$$

$$m_{\lambda\mu}(\vec{r}) = \mu_N\left[g_s\vec{s}(\vec{r}) + \tfrac{2}{\lambda+1}g_l\left(\vec{r}\times\vec{j}(\vec{r})\right)\right]\cdot\vec{\nabla}Q_{\lambda\mu}(\vec{r}),$$

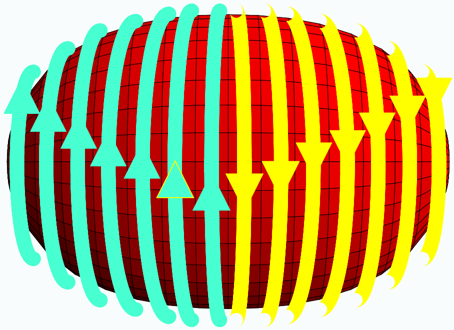
and e, g_s, and g_l are the elementary charge, and the spin and orbital gyromagnetic factors, respectively. The multipole functions (solid harmonics) have the standard form: $Q_{\lambda\mu}(\vec{r}) = r^\lambda Y_{\lambda\mu}(\theta,\phi)$.

Visualisation of the magnetic multipole moments in axial symmetry

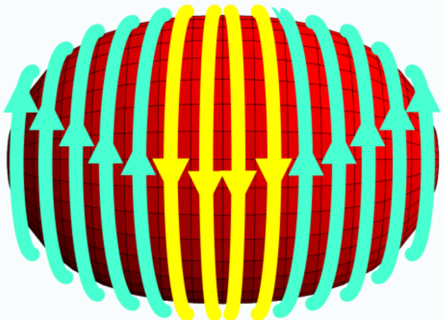
λ=1



λ=2



λ=3



Axial solid harmonics:

$\lambda\mu$	$Q_{\lambda\mu}$	$\nabla_z Q_{\lambda\mu}$
00	$\sqrt{\frac{1}{4\pi}}$	0
10	$\sqrt{\frac{3}{4\pi}}z$	$\sqrt{\frac{3}{4\pi}} \quad = \sqrt{3}Q_{00}$
20	$\sqrt{\frac{5}{16\pi}}\left(2z^2 - x^2 - y^2\right)$	$\sqrt{\frac{5}{\pi}}z \quad = \sqrt{\frac{20}{3}}Q_{10}$
30	$\sqrt{\frac{7}{16\pi}}\left(2z^3 - 3x^2z - 3y^2z\right)$	$\sqrt{\frac{7}{16\pi}}3\left(2z^2 - x^2 - y^2\right) \quad = \sqrt{\frac{63}{5}}Q_{20}$

Axial electric and magnetic-moment densities:

$$q_{\lambda 0}(r,\theta) = e\rho(r,\theta)Q_{\lambda 0}(r,\theta),$$

$$m_{\lambda 0}(r,\theta) = \mu_N\left[g_s s_z(r,\theta) + \tfrac{2}{\lambda+1}g_l\left(\vec{r}\times\vec{j}\right)_z(r,\theta)\right]\cdot\nabla_z Q_{\lambda 0}(r,\theta),$$

or

$$m_{\lambda 0}(r,\theta) = \mu_N\left[g_s s_z(r,\theta) + \tfrac{2}{\lambda+1}g_l I_z(r,\theta)\right]C_\lambda Q_{(\lambda-1)0}(r,\theta),$$

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HF + angular momentum projection (AMP)

The Hartree-Fock (HF) spin and current intrinsic densities read:

$$\vec{s}(\vec{r}) = \sum_{\sigma\sigma'} \vec{\sigma}_{\sigma'\sigma}\rho(\vec{r}\sigma, \vec{r}\sigma'), \quad \vec{j}(\vec{r}) = \frac{1}{2i}\sum_{\sigma}(\vec{\nabla} - \vec{\nabla}')\rho(\vec{r}\sigma, \vec{r}'\sigma),$$

where the one-body density matrix $\rho(\vec{r}\sigma, \vec{r}'\sigma')$ can be split into the core and odd-particle contributions:

$$\rho(\vec{r}\sigma, \vec{r}'\sigma') = \sum_{i=1}^{A-1} \psi_i(\vec{r}\sigma)\psi_i^*(\vec{r}'\sigma') + \psi_{\text{odd}}(\vec{r}\sigma)\psi_{\text{odd}}^*(\vec{r}'\sigma'),$$

and where $\psi(\vec{r}\sigma)$ are the self-consistent single-particle wave functions of occupied states. The HF wave function of an odd system $|\Phi\rangle = |\Phi^{\text{core}}\rangle \otimes |\psi^{\text{odd}}\rangle = \sum_I |\Psi_I\rangle$ has the conserved-angular-momentum components:

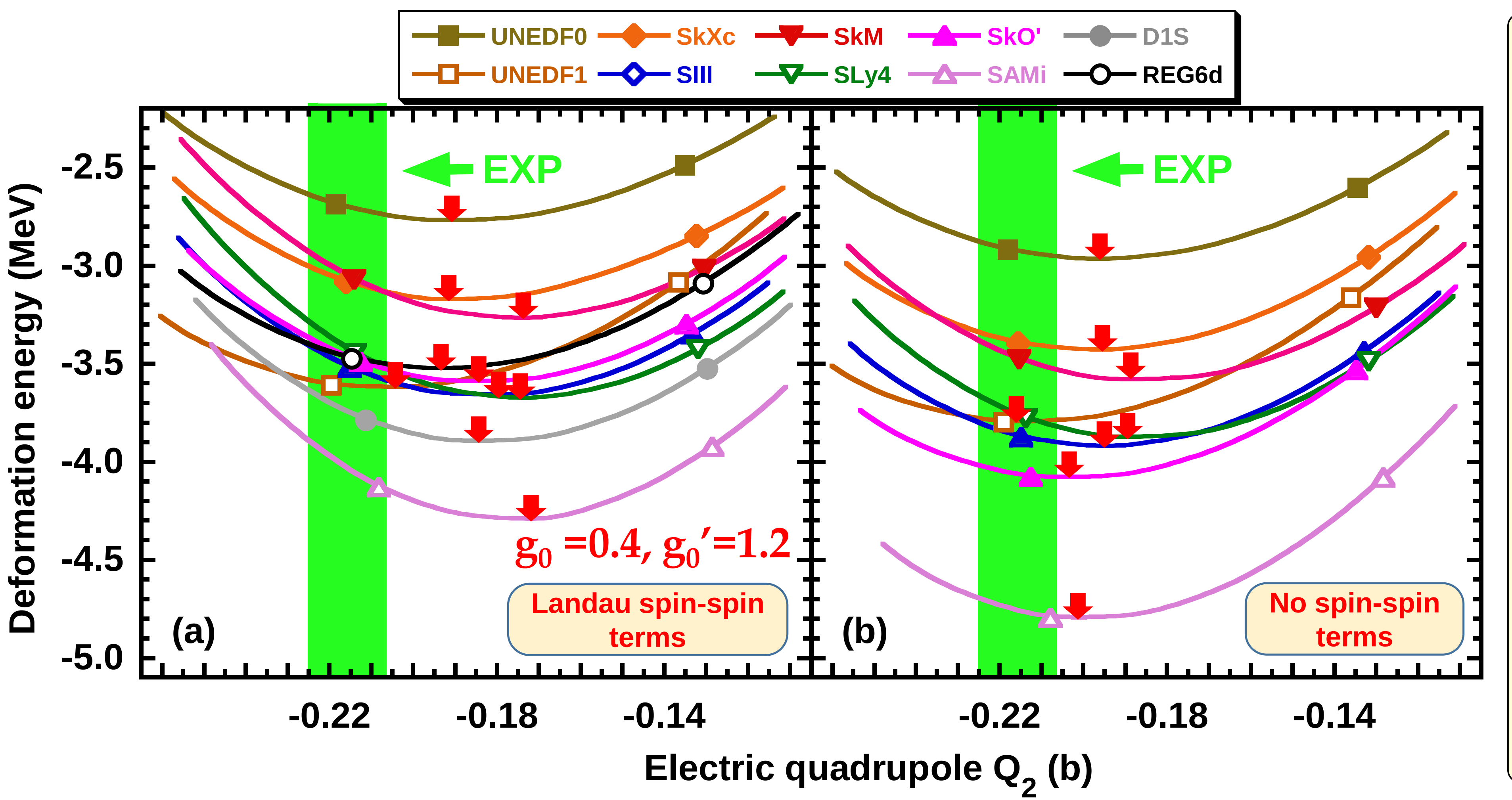
$$|\Psi_I\rangle = \sum_{J=0,2,4,\ldots} \; \sum_{j=K,K+2,K+4,\ldots} \left[|\Psi_J^{\text{core}}\rangle|\psi_j^{\text{odd}}\rangle\right]_I ,$$

In ^{45}Sc, the angular-momentum projected ground state can be presented as:

$$\begin{aligned} |\Psi_{7/2}\rangle &= \left[|\Psi_0^{\text{core}}\rangle|\psi_{7/2}^{\text{odd}}\rangle\right]_{7/2} + \left[|\Psi_2^{\text{core}}\rangle|\psi_{7/2}^{\text{odd}}\rangle\right]_{7/2} \\ &+ \left[|\Psi_2^{\text{core}}\rangle|\psi_{11/2}^{\text{odd}}\rangle\right]_{7/2} + \left[|\Psi_4^{\text{core}}\rangle|\psi_{7/2}^{\text{odd}}\rangle\right]_{7/2} + \ldots \end{aligned}$$

The first term represents a spherical core coupled to the spherical $j = 7/2$ wave function of the odd particle. The second term represents the lowest-order coupling of the odd-particle to the lowest $J = 2$ state of the core.

HF+AMP, deformation energies in ^{45}Sc

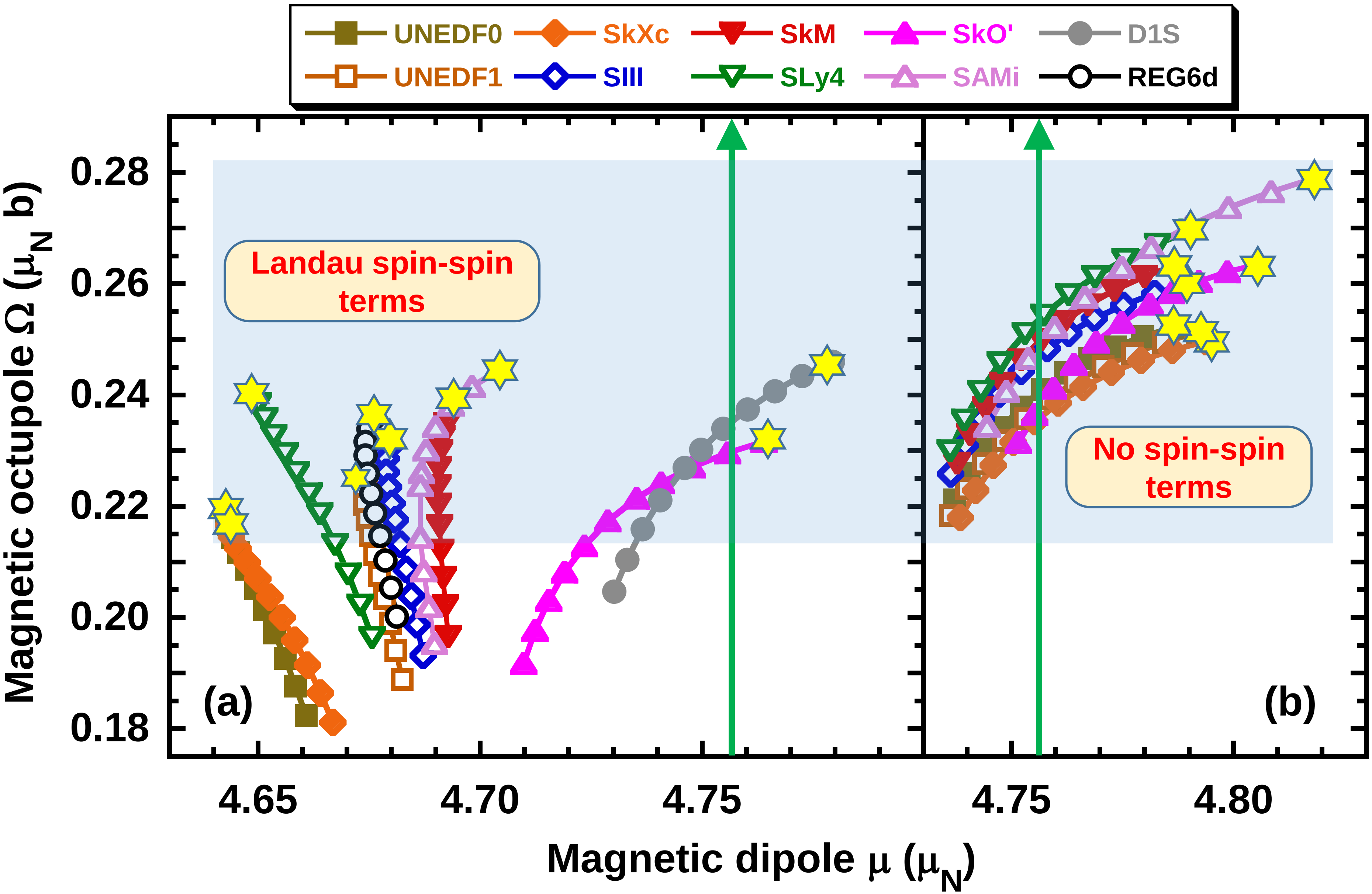


R. P. de Groote *et al.*, arXiv:2005.00414

isoscalar isovector Landau parameters g_0 & g_0'

$$V(\vec{r}_1,\vec{r}_2,\vec{r}_3,\vec{r}_4) = N_0\Big[g_0(\sigma_1\cdot\sigma_2) + g_0'(\sigma_1\cdot\sigma_2)(\tau_1\cdot\tau_2)\Big]\delta(\vec{r}_1-\vec{r}_2)\delta(\vec{r}_1-\vec{r}_3)\delta(\vec{r}_2-\vec{r}_4)$$

HF+AMP, magnetic moments in ^{45}Sc



R. P. de Groote *et al.*, arXiv:2005.00414

Ab initio magnetic moments in light nuclei

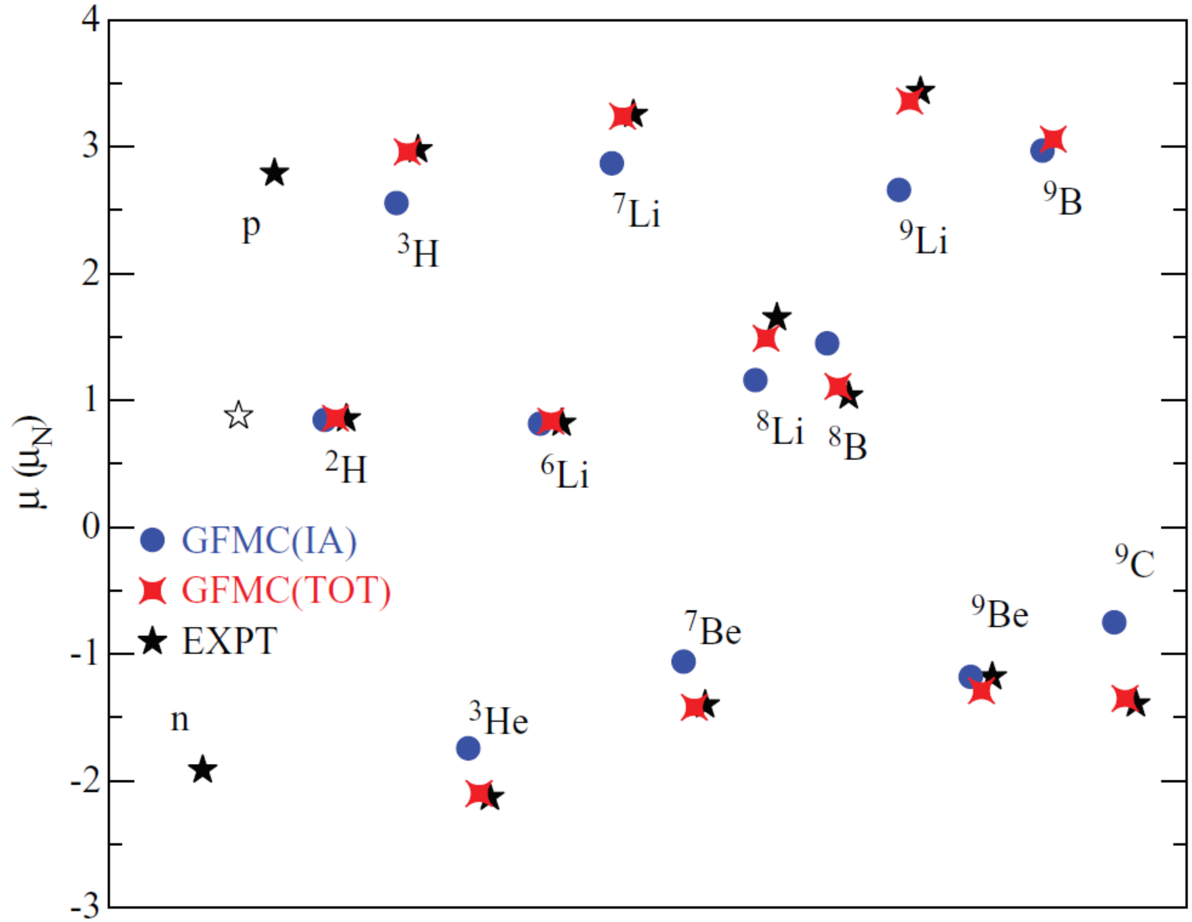


FIG. 4. (Color online) Magnetic moments in nuclear magnetons for $A \leqslant 9$ nuclei. Black stars indicate the experimental values [35–37], while blue dots (red diamonds) represent GFMC calculations which include the IA one-body EM current (total χEFT current up to N3LO). Predictions are for nuclei with $A > 3$.

S. Pastore *et al.*, Phys. Rev. C 87, 035503 (2013)

Conclusions

1. Ground-state and isomeric magnetic moments are known in hundreds of odd and odd-odd nuclei, measured by atomic spectroscopic methods up to a **very high precision**.
2. In the standard shell-model calculations, agreement with data is achieved by using the concept of **effective g-factors**.
3. In the nuclear DFT calculations, magnetic moment have been up to now **rarely considered**.
4. Poorly known **time-odd sector** of the nuclear DFT crucially influences the magnetic moments.
5. **Adjustments of the nuclear DFT coupling constants** to data should take the magnetic moments into account.

Thank you

NucMagMom Collaboration (est. 2017)

- **Michael Bender, Lyon**
- **Witek Nazarewicz, Mengzhi Chen, MSU**
- **J.D., Jérémy Bonnard, Paolo Sassarini, Alessandro Pastore, York**
- **.... all wishing to join are welcome**

Literature

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- **N.J. Stone, At. Data and Nucl. Data Tables 90 (2005) 75–176.**
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Basics

The magnetic operator $\bar{\mu}$ is a one-body operator and the magnetic dipole moment μ is the expectation value of $\bar{\mu}_z$. The M1 operator acting on a composed state $|\mathrm{Im}\rangle$ can then be written as the sum of single particle M1 operators $\bar{\mu}_z(j)$ acting each on an individual valence nucleon with total momentum j:

$\mu = g_L\, \mathbf{L} + g_s\, \mathbf{s}$

$$\mu(I) \equiv \left\langle I(j_1, j_2, \ldots, j_n), m = I \left| \sum_{i=1}^{n} \bar{\mu}_z(i) \right| I(j_1, j_2, \ldots, j_n), m = I \right\rangle \tag{2.1}$$

The single particle magnetic moment $\mu(j)$ for a valence nucleon around a doubly magic core is uniquely defined by the quantum numbers l and j of the occupied single particle orbit [22]:

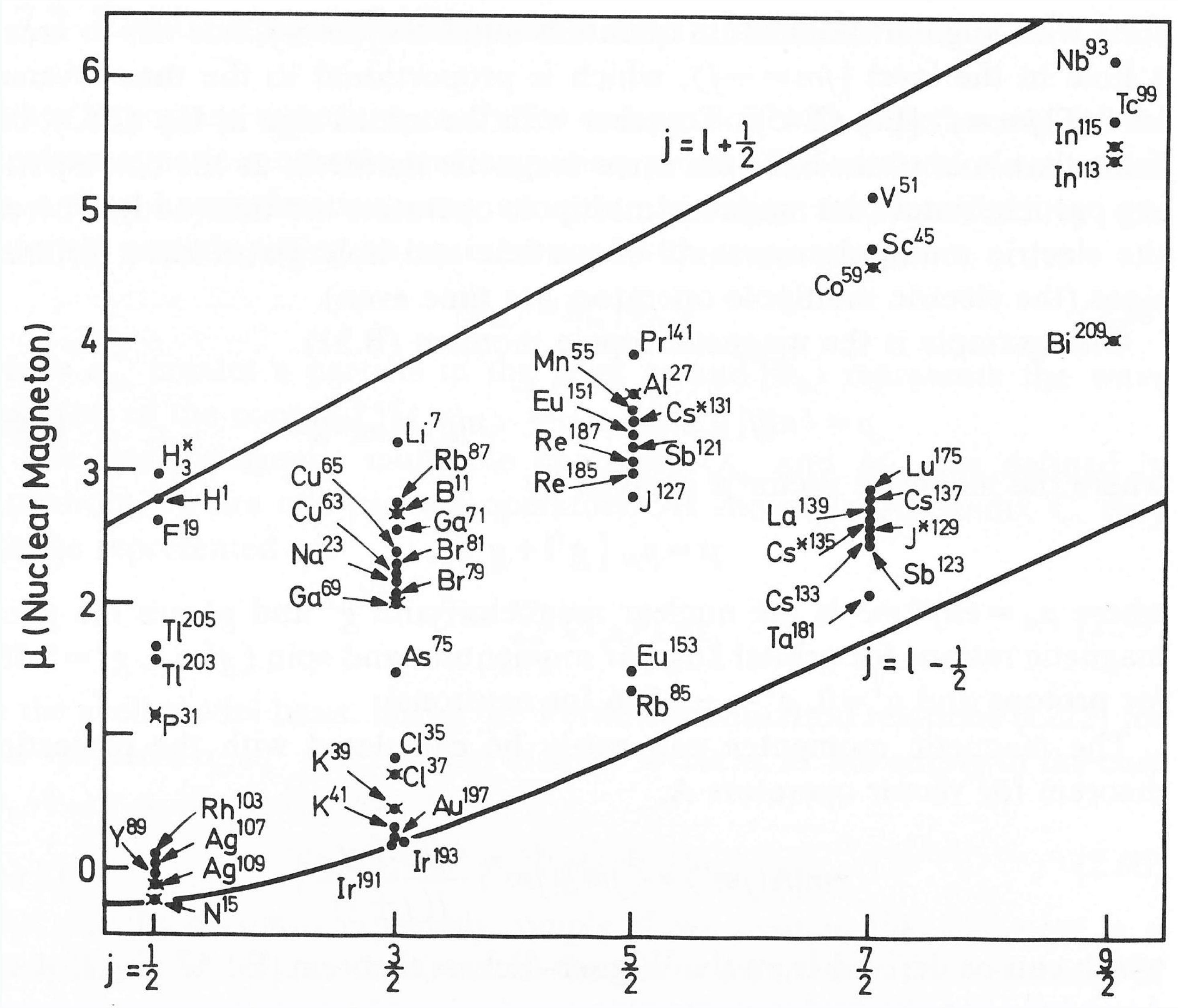
$$\text{for an odd proton:} \begin{cases} \mu = j - \frac{1}{2} + \mu_{\mathrm{p}} & \text{for } j = l + \frac{1}{2} \\ \mu = \dfrac{j}{j+1}\left(j + \dfrac{3}{2} - \mu_{\mathrm{p}}\right) & \text{for } j = l - \frac{1}{2} \end{cases} \tag{2.2}$$

$$\text{for an odd neutron:} \begin{cases} \mu = \mu_{\mathrm{n}} & \text{for } j = l + \frac{1}{2} \\ \mu = -\dfrac{j}{j+1}\mu_{\mathrm{n}} & \text{for } j = l - \frac{1}{2} \end{cases} \tag{2.3}$$

These single particle moments calculated using the free proton and free neutron moments ($\mu_{\mathrm{p}} = +2.793$, $\mu_{\mathrm{n}} = -1.913$) are called the Schmidt moments. In a nucleus, the magnetic

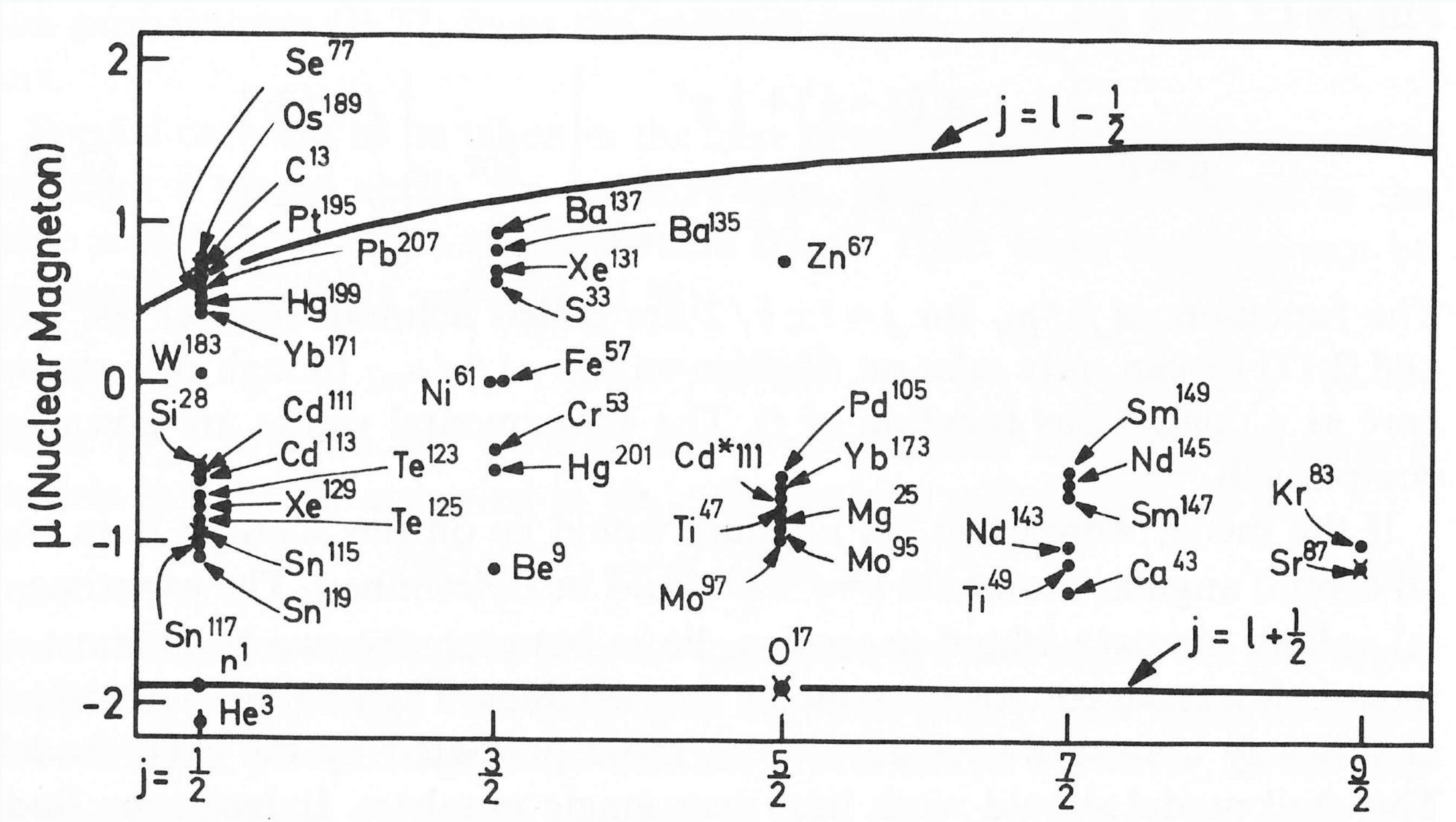
Gerda Neyens, Rep. Prog. Phys. 66 (2003) 633–689

Experiment



M.G. Mayer and J.H.D. Jensen, *Elementary Theory of Nuclear Shell Structure*, (Wiley, New York, 1955)

Experiment



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