Polarisation Vision

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THE ROYAL SOCIETY

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Overview

- Background
- Single View Polarization Vision
- Multiple View Polarization Vision
- Patch Merging
- BRDF Estimation
- Results
- Conclusions

Shape-from-shading

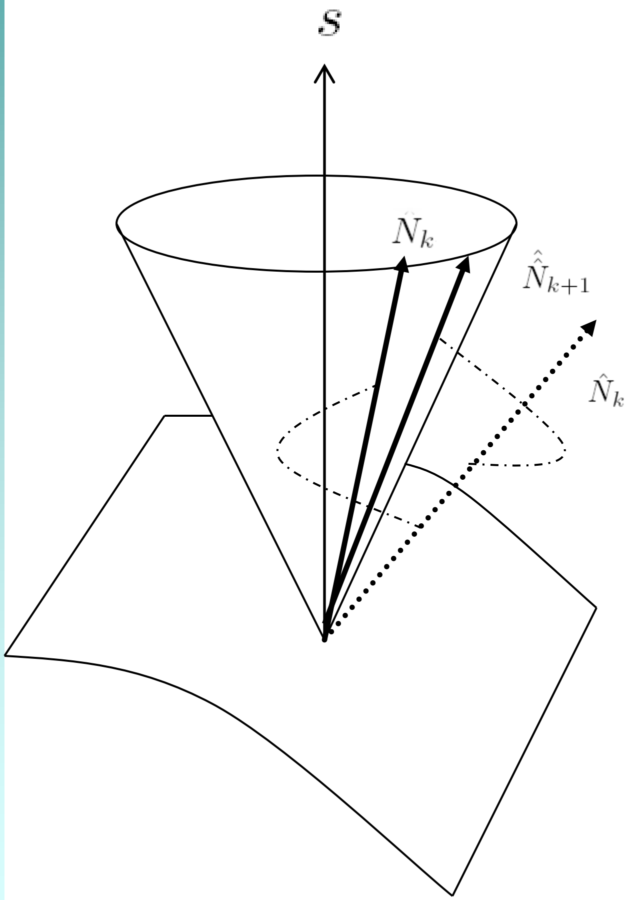
Recover surface normals and hence surface height from observed variations in image brightness,

If surface reflectance is Lambertian, then surface normal lies on a cone whose axis is the light source direction and whose opening angle is the inverse cosine of the normalised image brightness,

Hence zenith angles of surface normals are determined by Lambert's law, azimuth angles determined by boundary conditions and smoothness constraints.

Geometric SFS

(Worthington and Hancock '99)



Surface normal must fall on a cone whose axis is light source direction and whose opening angle is determined by image brightness.

$$I = n.s = \cos\theta$$

Polarisation

Method breaks down for surfaces which are non-Lambertian, and this includes those that are shiny (exhibit specularities) and those that are rough.

Aim in this talk is to use the Fresnel theory to show that polarisation measurements can be used to determine the zenith and azimuth angles for shiny surfaces.

We make use of diffuse polarisation. Hence, the incoming light is unpolarised but develops a spontaneous polarisation due to interaction with the surface.

Degree of diffuse polarisation determines the surface normal zenith angle, and the phase-angle the azimuth angle (subject to ambiguity)

Publications relevant to work

- Use diffuse polarisation measurements to estimate surface orientation (IEEE TIP 06).

- Extend to multiple views to resolve ambiguities and extend object coverage (PAMI 07).

- Use method to estimate BRDF’s for surfaces composed of different materials (CVIU 08).

- Explore whether multiple images taken with fixed camera direction and varying light source direction (i.e. photometric stereo) can he used to resolve azimuth angle ambiguity (CAIP 07)

Polarization Vision and Applications

Shape from polarization

Polarization vision	Wolff and Boult TPAMI ’91
Shape recovery	Miyazaki et al ICCV ’03, TPAMI ’04
	Drbohlav and Šára SPIE ’99
	Rahmann and Canterakis CVPR ’01

Other uses of polarization

Reflection components	Umeyama TPAMI ’04
Photometric stereo	Drbohlav and Šára ICCV ’01
Range scanning	Clark, Trucco and Wolff IVC ’97
Marine vision	Schechner and Karpel CVPR ’03
BRDF estimation	Shibata et al SPIE ’05
Segmentation / classification	Chen and Wolff IJCV ’98

An end user

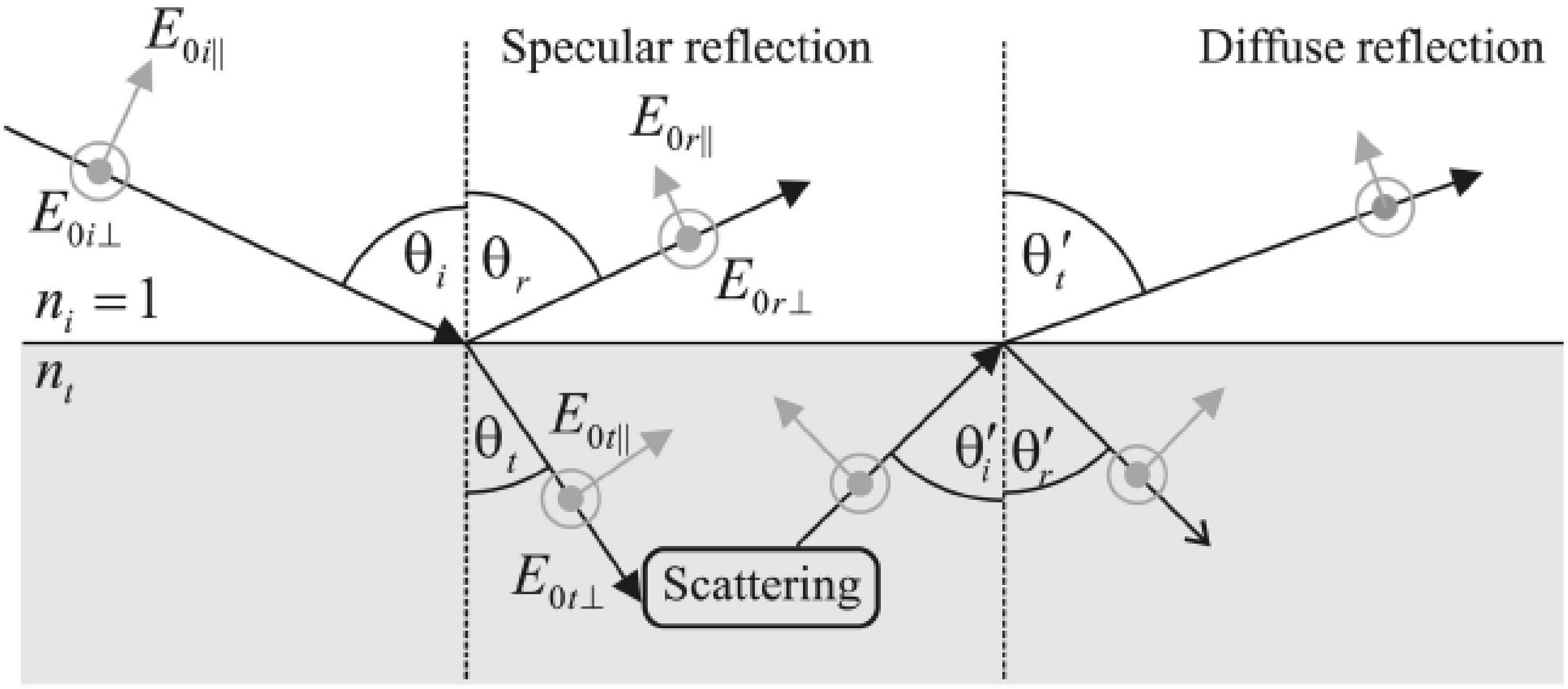
The mantis shrimp

Theoretical background

Fresnel theory

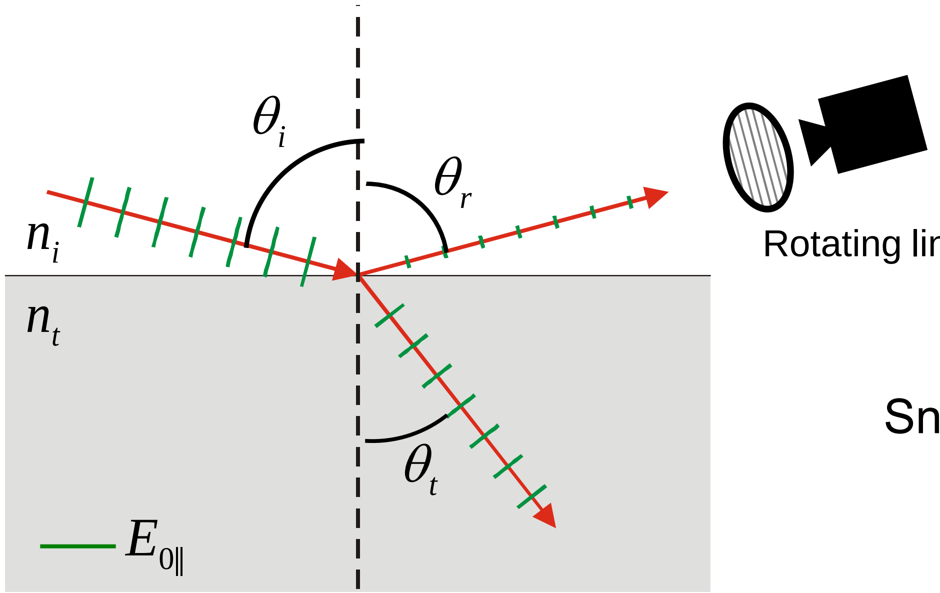
Augustin-Jean Fresnel (1788-1827)

Basic concepts



Theory: Physical Origins of Polarization by Reflection

Specular reflection



Snell's Law

$$n_i \sin\theta_i = n_t \sin\theta_t$$

Fresnel Coefficients

Perp to incidence plane

$$r_\perp \equiv \frac{E_{0r\perp}}{E_{0i\perp}} = \frac{n_i\cos\theta_i - n_t\cos\theta_t}{n_i\cos\theta_i + n_t\cos\theta_t} \qquad R_\perp = r_\perp^{\,2}$$

Parallel to incidence plane

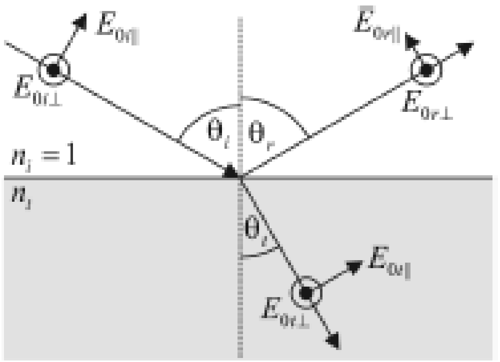
$$r_\parallel \equiv \frac{E_{0r\parallel}}{E_{0i\parallel}} = \frac{n_t\cos\theta_i - n_i\cos\theta_t}{n_t\cos\theta_i + n_i\cos\theta_t} \qquad R_\parallel = r_\parallel^{\,2}$$

Origins of polarisation

Occurs when light is reflected from boundary between layers of different refractive index

Polarisation for specular reflection

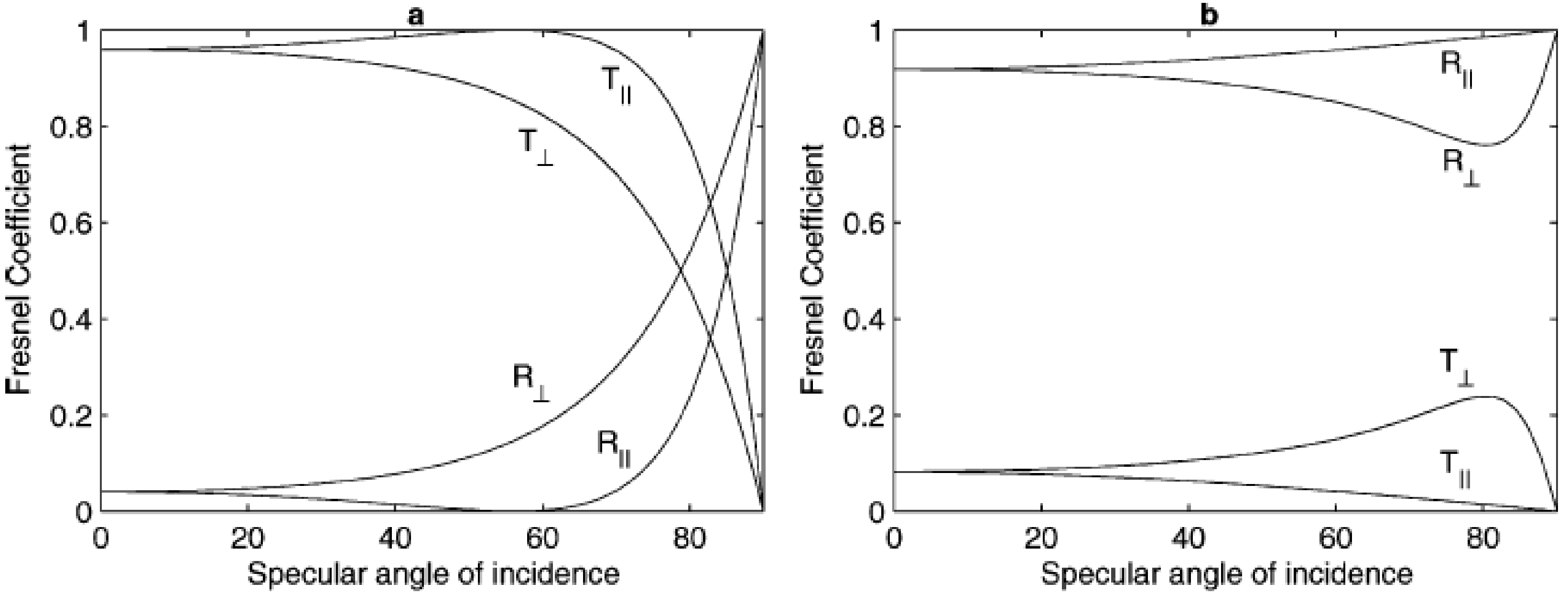
Defined in terms of reflection coefficients for different planes of polarisation.



$$\rho_s = \frac{R_\perp(n,\theta_i) - R_\parallel(n,\theta_i)}{R_\perp(n,\theta_i) + R_\parallel(n,\theta_i)}$$

$$\rho_s = \frac{2\sin^2\theta\cos\theta\sqrt{n^2 - \sin^2\theta}}{n^2 - \sin^2\theta - n^2\sin^2\theta + 2\sin^2\theta}$$

Behaviour

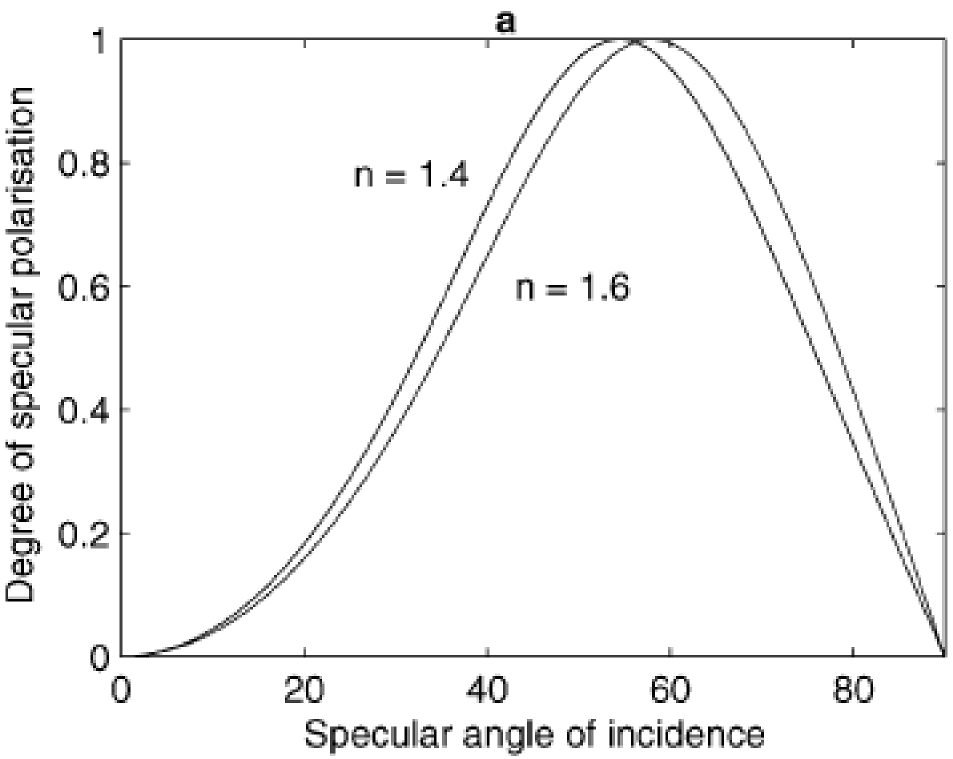


(a) (b)

Dielectric n=1.7

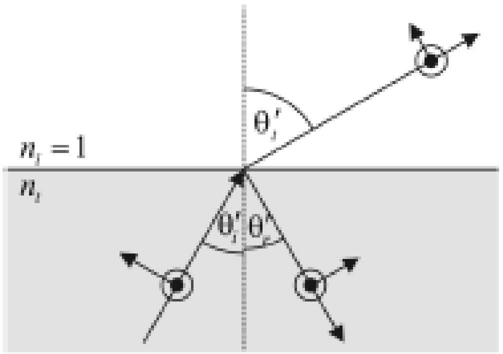
Metal n=0.8, k=6.

Specular polarisation versus incidence angle



Because of Brewster angle. For a measured polarisation there are two possible incidence angles

Polarisation for diffuse reflection



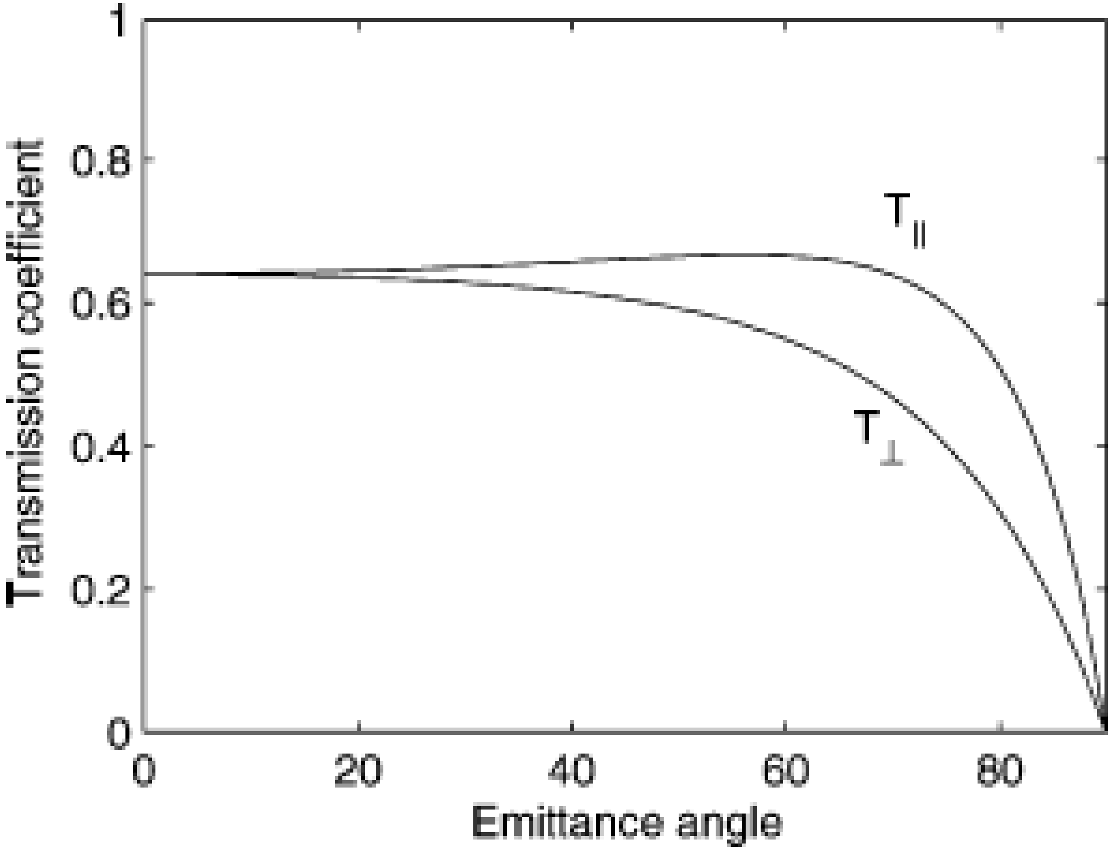
Defined in terms of transmission rather than reflection: T=1-R

$$\rho_d = \frac{T_\parallel(1/n,\theta_i') - T_\perp(1/n,\theta_i')}{T_\parallel(1/n,\theta_i') + T_\perp(1/n,\theta_i')} = \frac{R_\perp(1/n,\theta_i') - R_\parallel(1/n,\theta_i')}{2 - R_\perp(1/n,\theta_i') - R_\parallel(1/n,\theta_i')}$$

Use Snell’s law to re-express in terms of emittance angle

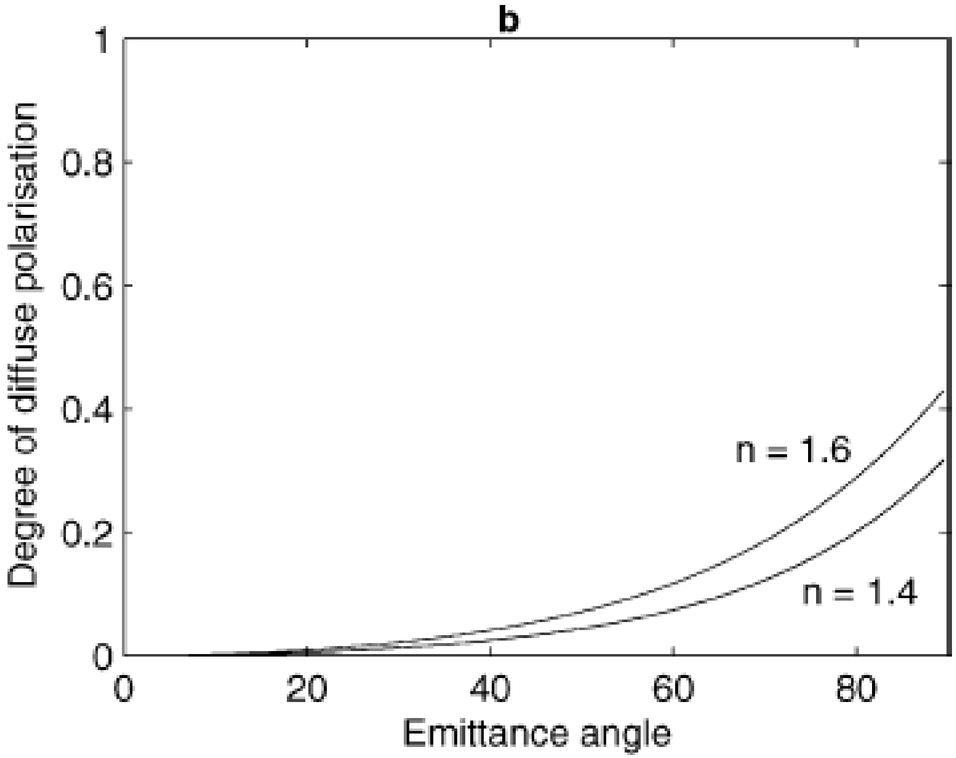
$$\rho_d = \frac{(n - 1/n^2)\sin^2\theta}{2 + 2n^2 - (n - 1/n^2)\sin^2\theta + 4\cos\theta\sqrt{n^2 - \sin^2\theta}}$$

Behaviour



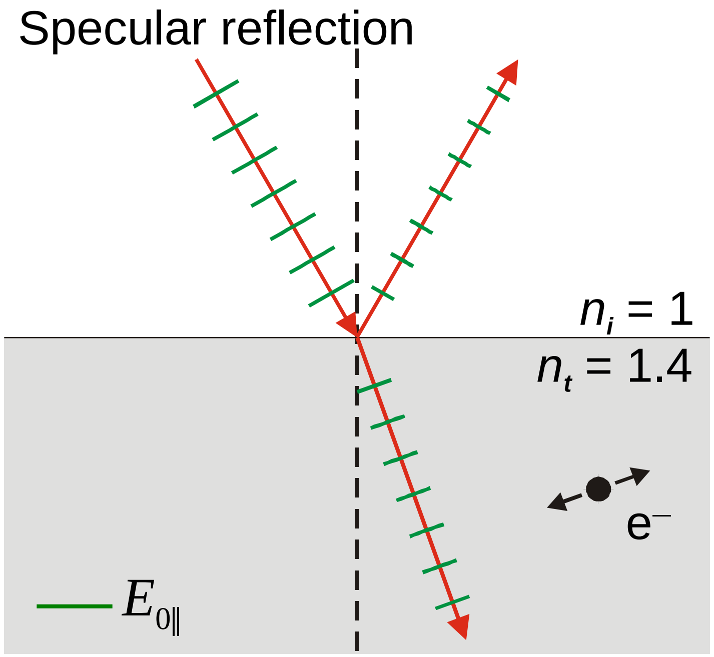
1
0.8
0.6
0.4
0.2
0
Transmission coefficient
0
20
40
60
80
Emittance angle
$T_{||}$
$T_{\perp}$

Diffuse polarisation versus emittance angle



No Brewster angle for diffuse polarisation. Single measurement of polarisation gives a single emittance angle.

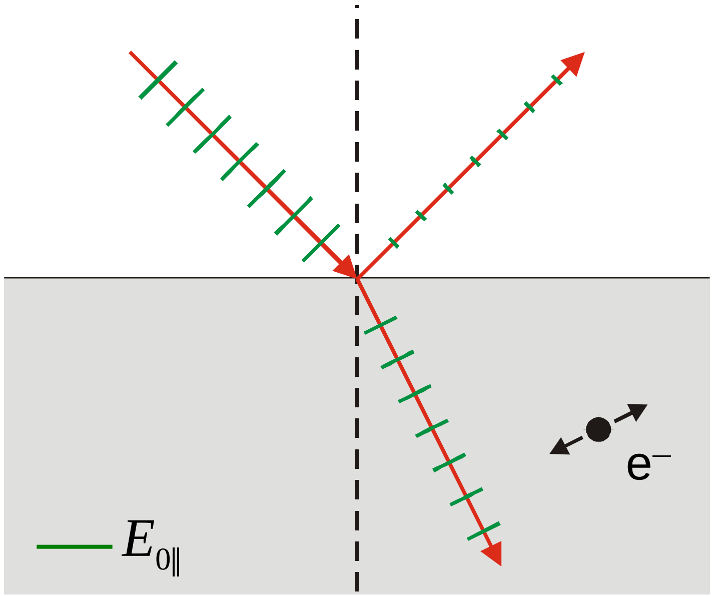
Theory: Polarization by Reflection



Low polarization

Theory: Polarization by Reflection

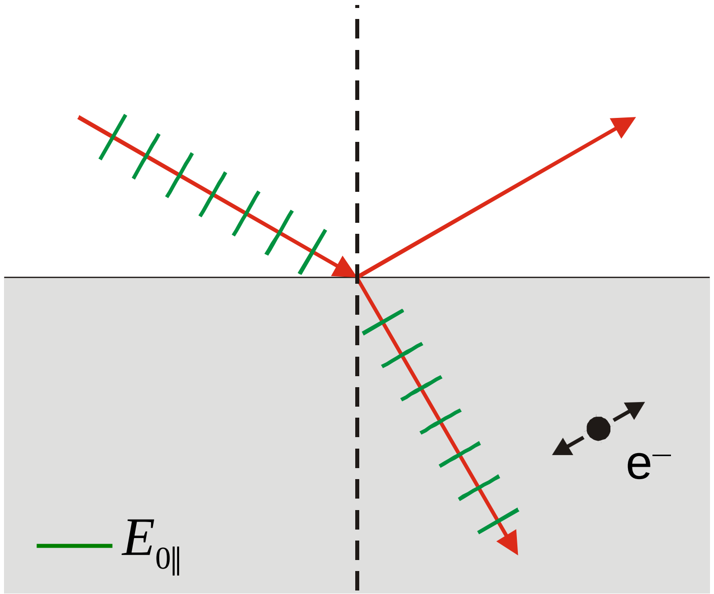
Specular reflection



High polarization

Theory: Polarization by Reflection

Specular reflection

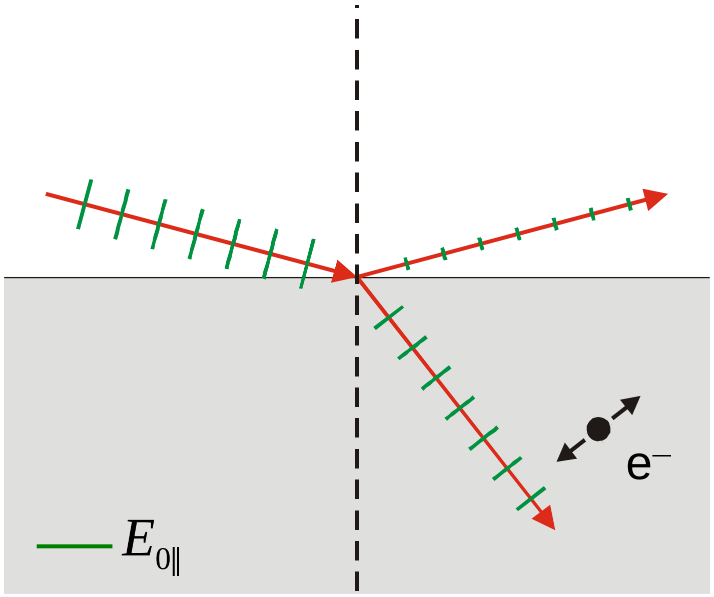


Complete polarization:
Brewster Angle

Reflected light totally extinguished by rotating polariser.

Theory: Polarization by Reflection

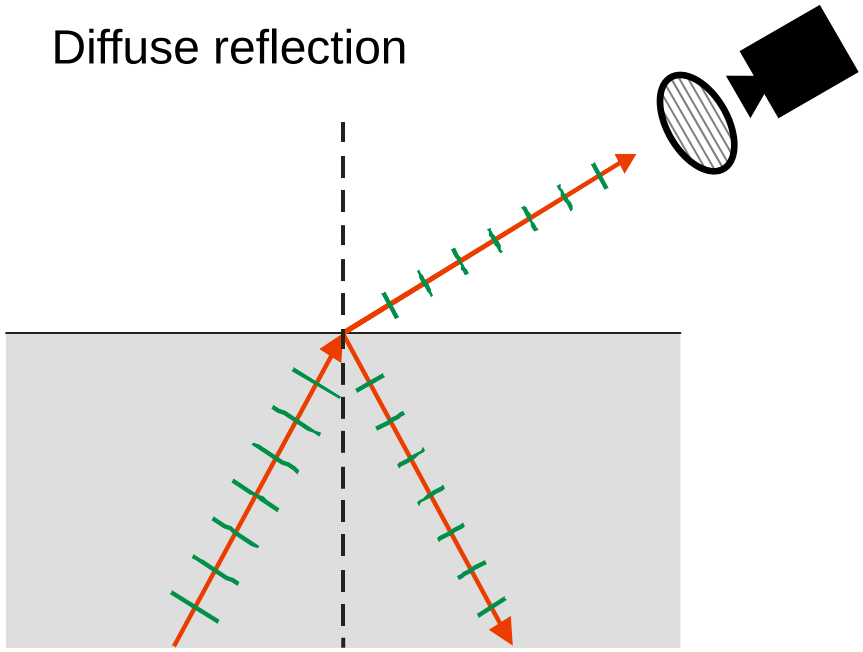
Specular reflection



High polarization

Theory: Shape from Diffuse Polarization

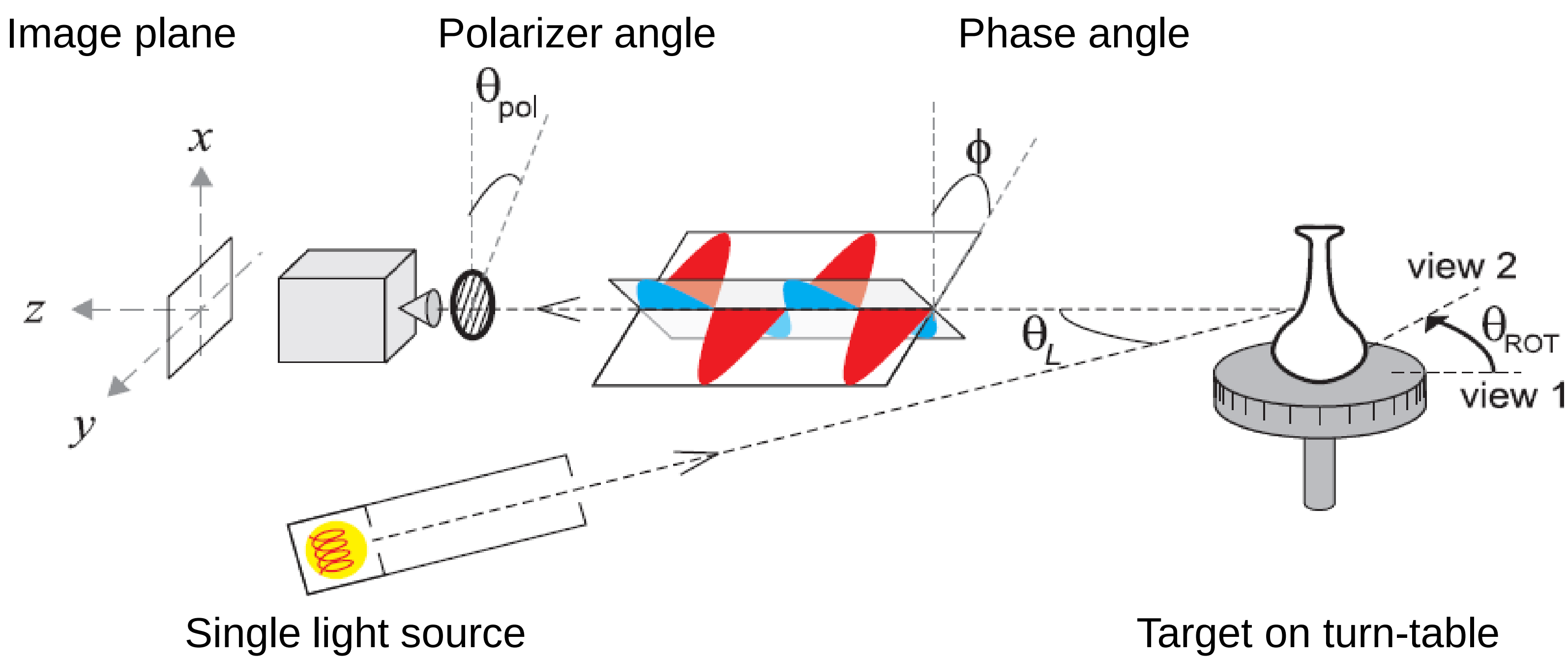
Diffuse reflection



Diffuse component emerges
after subsurface scattering

Polarisation Camera

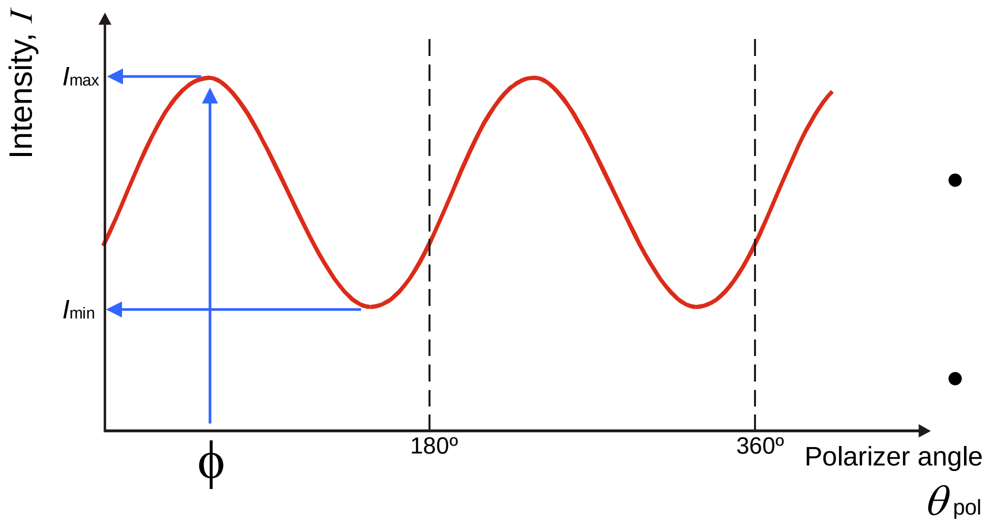
1. Acquire polarization images with light souce, camera and object fixed while polarizer rotates



Note: incident light is unpolarised.

Polarisation measurments

Measured intensity variation



Degree of polarization

$$\rho = \frac{I_{\max} - I_{\min}}{I_{\max} + I_{\min}}$$

- Rotate polariser and measure brightness at each pixel with camera, light source and object fixed.
- Brightness varies sinusoidally with polariser angle.
- Fit to recover maximum and minimum brightness together with phase of sinusoid at each pixel.
- Compute polarisation from max and min brightnesses.

Polarisation Image

- Composed of brightness, phase and polarisation

Brightness

Phase

Polarisation

Single view shape reconstruction

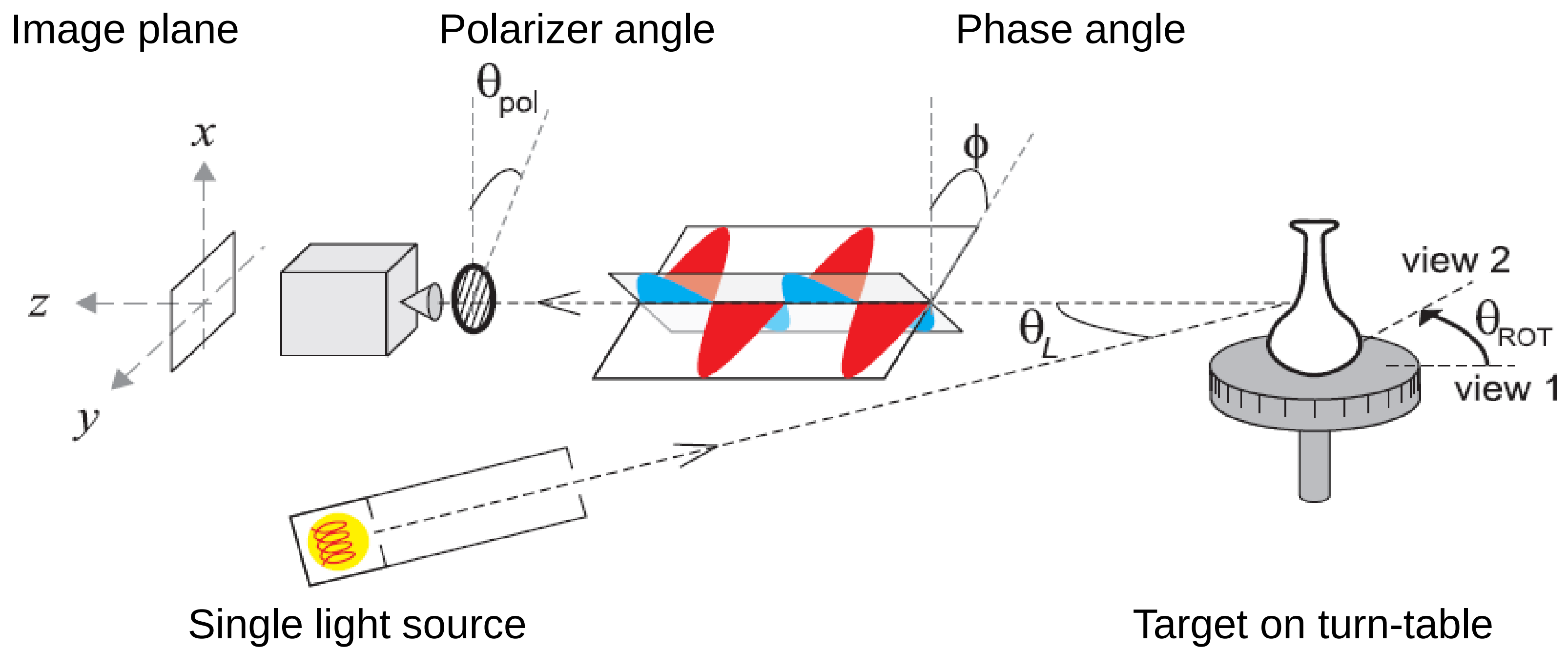
Use estimates of zenith and azimuth angles to recover surface normals. Reconstruct object shape using surface integration.

Single View Shape Recovery: Overview

1. Acquire polarization images
2. Estimate zenith angles from degree of polarization
3. Ambiguously estimate azimuth angles
4. Disambiguate azimuth angles
5. Integrate normals using Frankot-Chellappa method [TPAMI ’88]

Single View Vision: Apparatus

1. Acquire polarization images



Single View Vision: Method

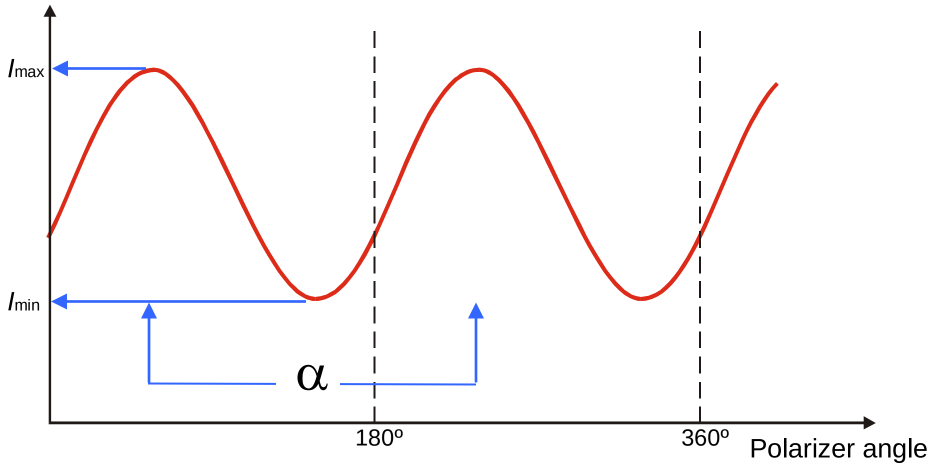
A: Estimate zenith angles from degree of polarization

$$\rho_d = \frac{(n - 1/n)^2 \sin^2\theta}{2 + 2n^2 - (n + 1/n)^2 \sin^2\theta + 4\cos\theta\sqrt{n^2 - \sin^2\theta}}$$

Single real solution since polarisation increases monotonically with emittance angle. i.e. there is no Brewster angle for diffuse polarisation.

Single View Vision: Method

B: Ambiguously estimate azimuth angles from measured phase



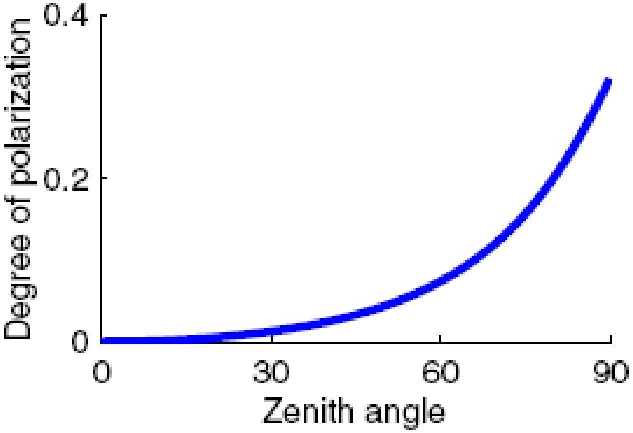
$$\alpha = \begin{cases} \phi \\ \phi + \pi \end{cases}$$

Azimuth angle of surface normal is orientation of projection of surface normal onto image plane. Light is reflected most efficiently when polarised parallel to plane containing surface normal and reflected ray. Hence, phase of polarised light is equivalent to azimuth angle of surface normal up to an ambiguity of 180 degrees.

Disambiguation

- On boundary select azimuth angle that is closest to that of occluding boundary normal.
- Propagate constraint as brush-fire into interior of object.
- For small zenith angles allow aburpt changes of azimuth angle.

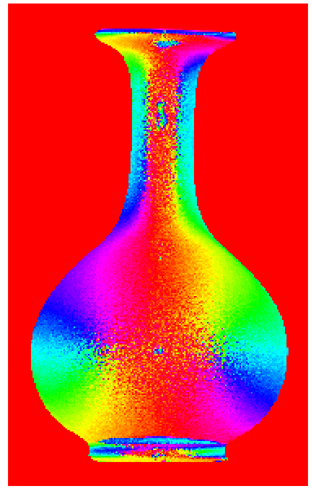
- Diffuse polarisation solved for surface normal zenith angle (unambiguously)

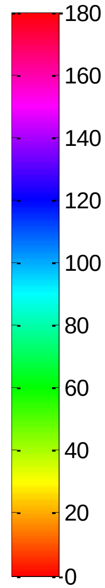


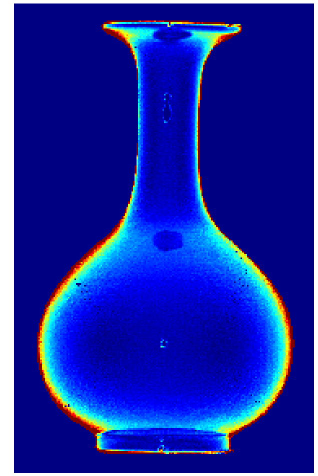
- Analogous to shape-from-shading, where Lambert's law allows zenith angle to be determined from measured image brightness

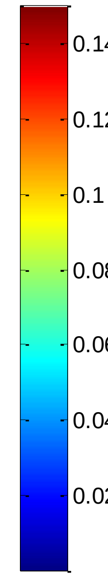
$$L = n.s = \cos\theta$$

Single View Vision: Method





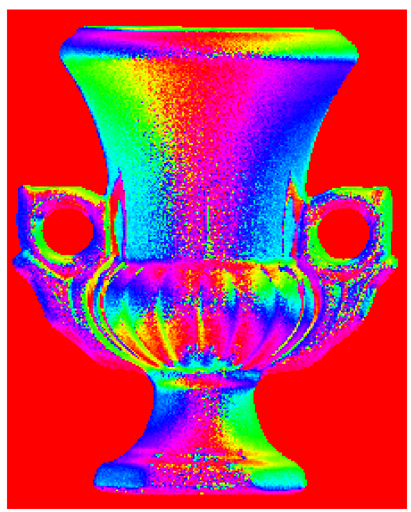


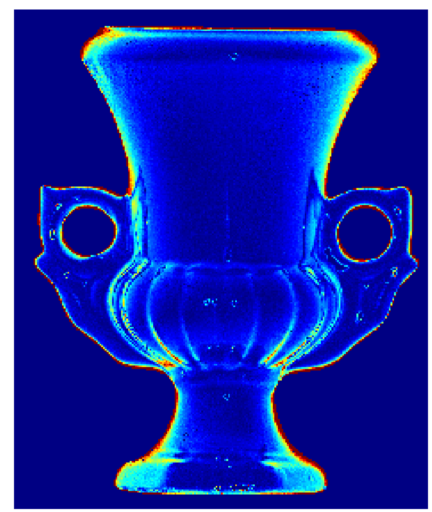


Intensity, I

Phase, ϕ

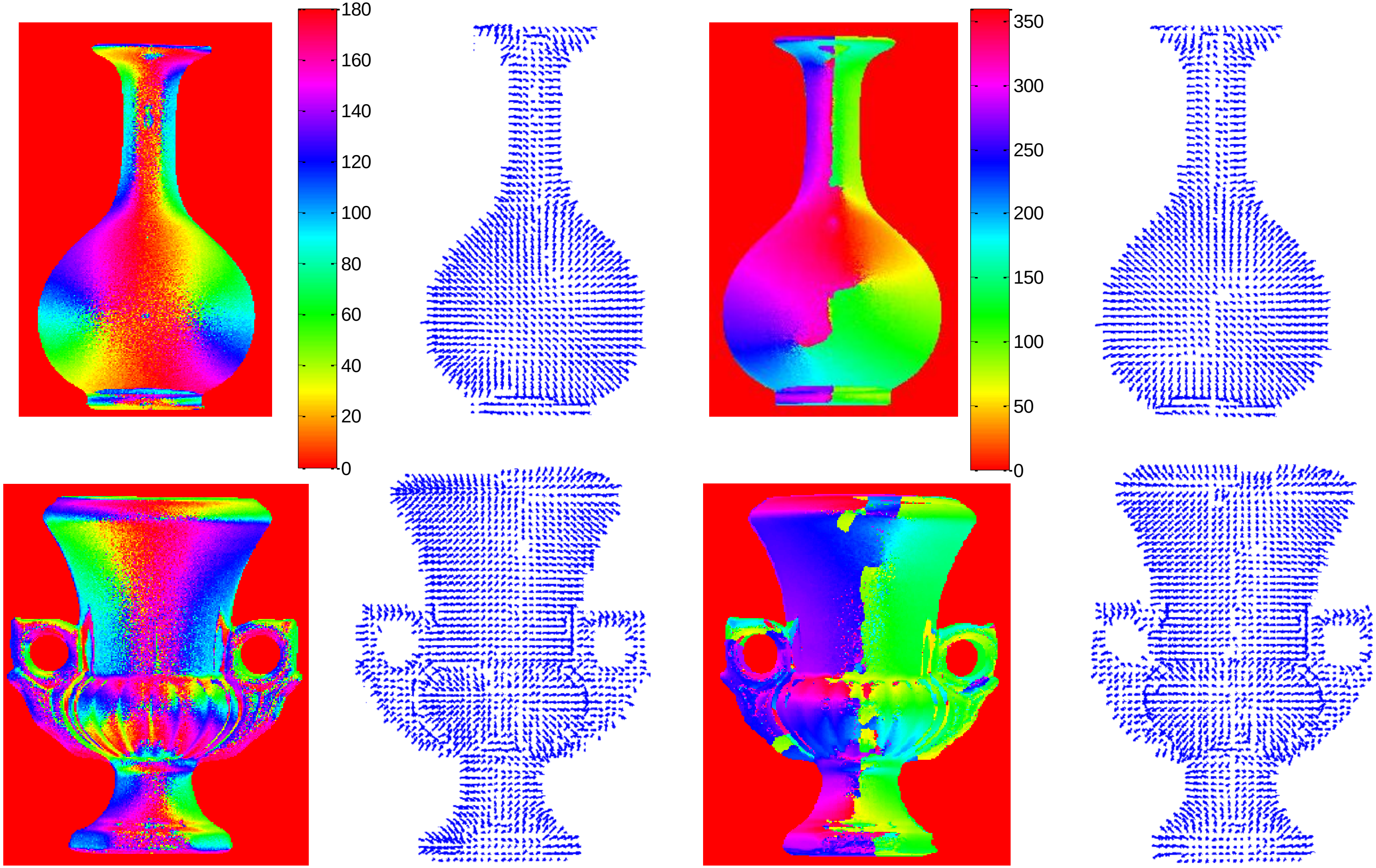
Degree of polarization, ρ



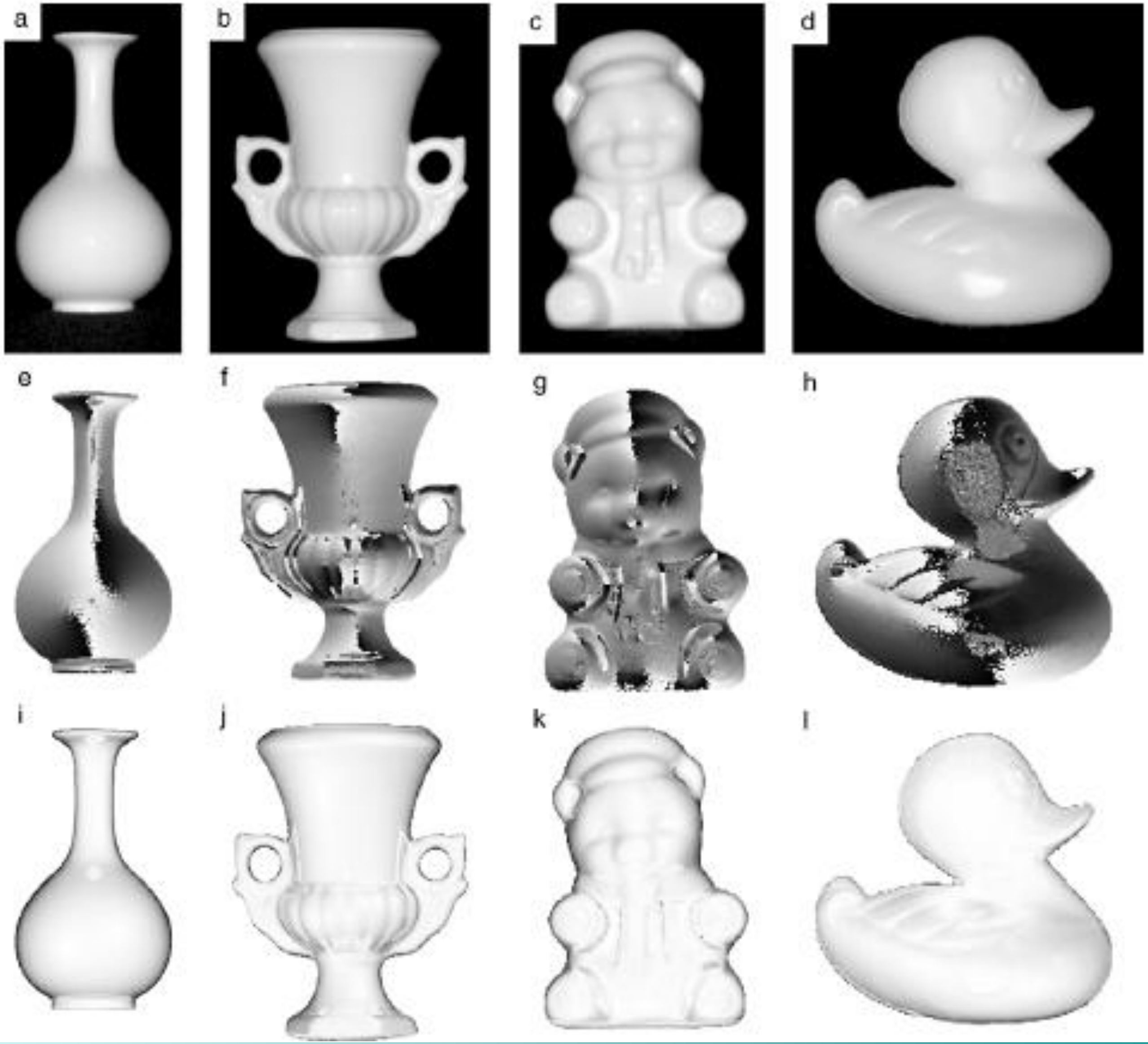


Single View Vision: Method

4. Disambiguate azimuth angles

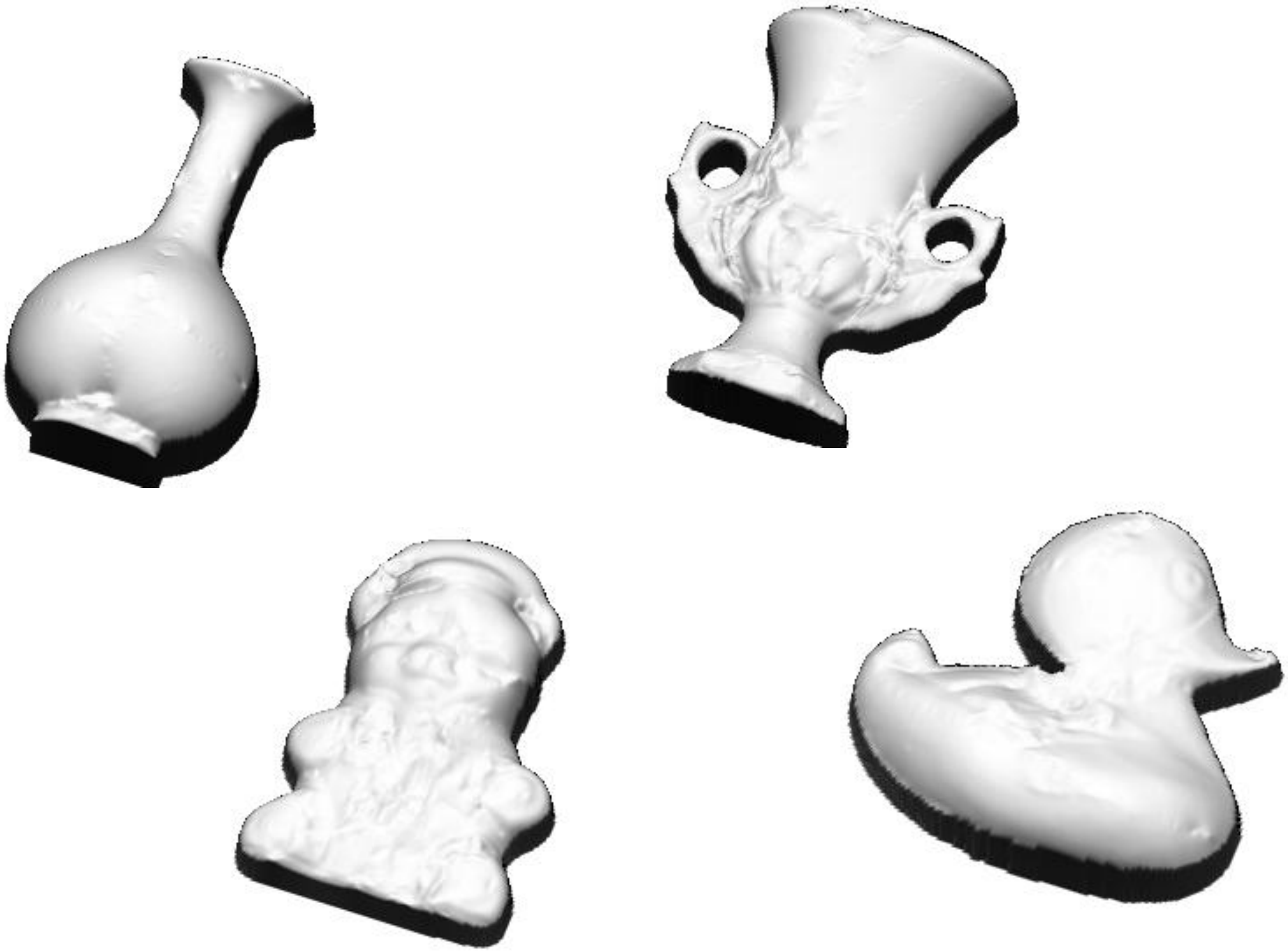


Examples

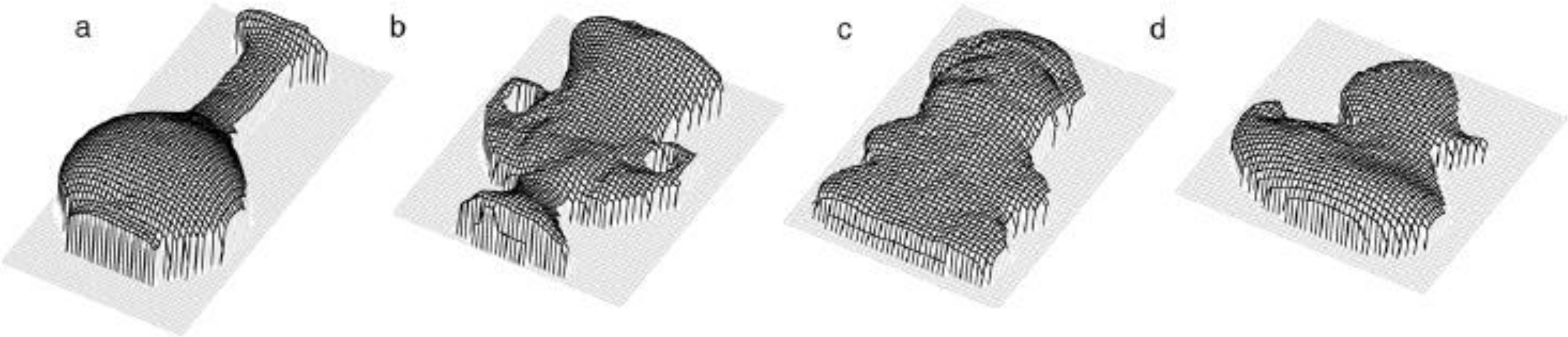


Results

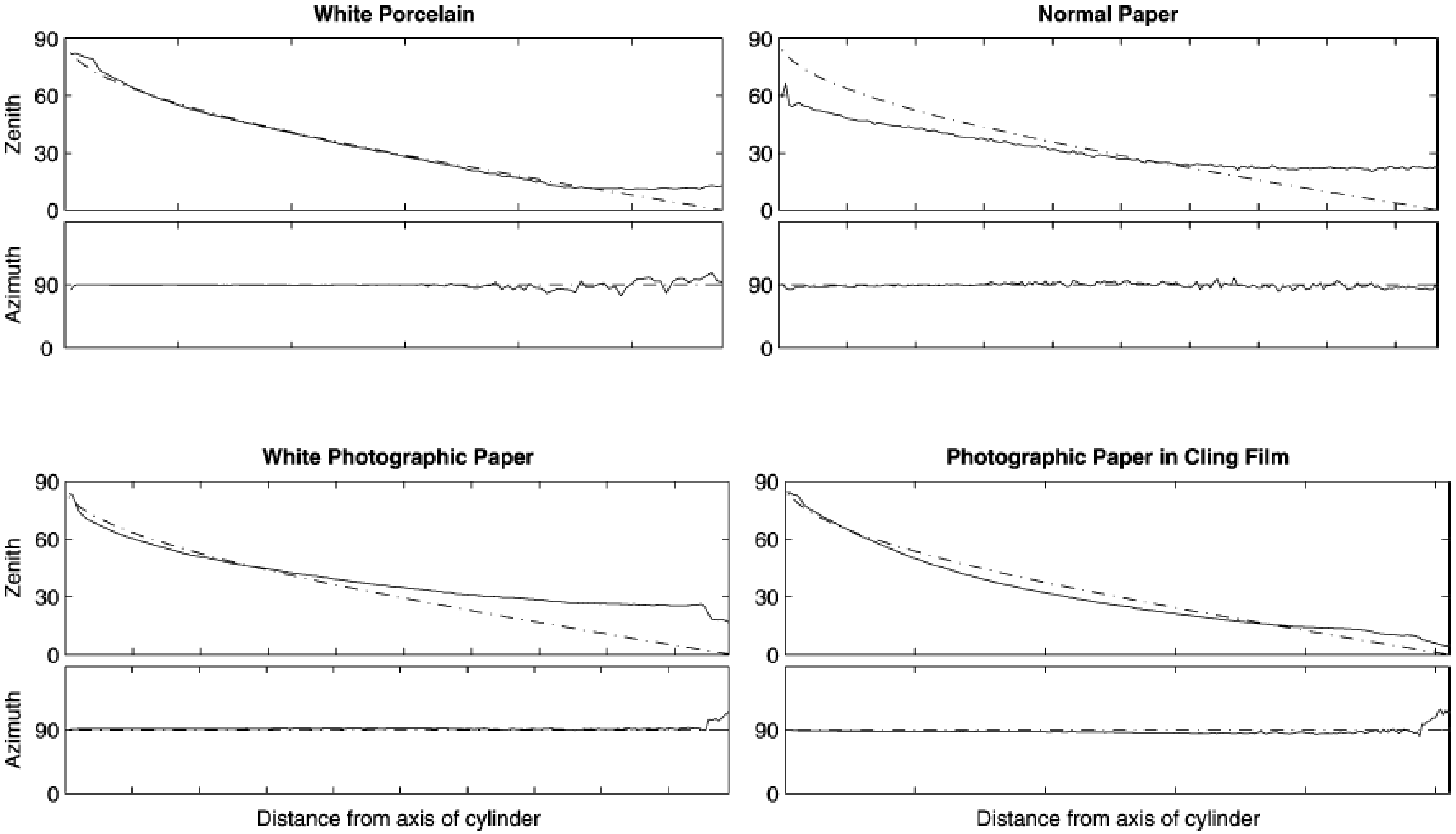
5. Integrate surface normals



Height functions

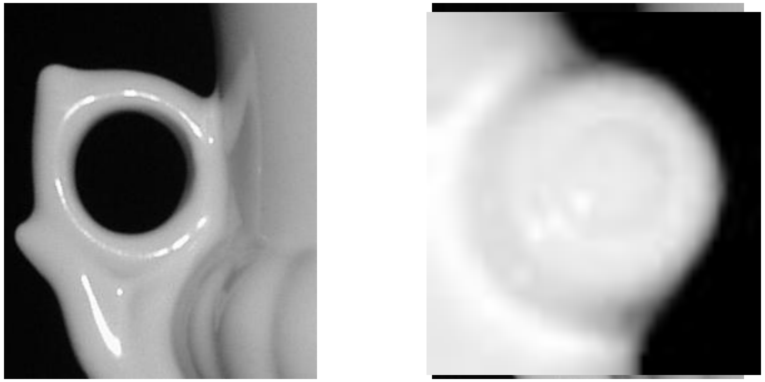


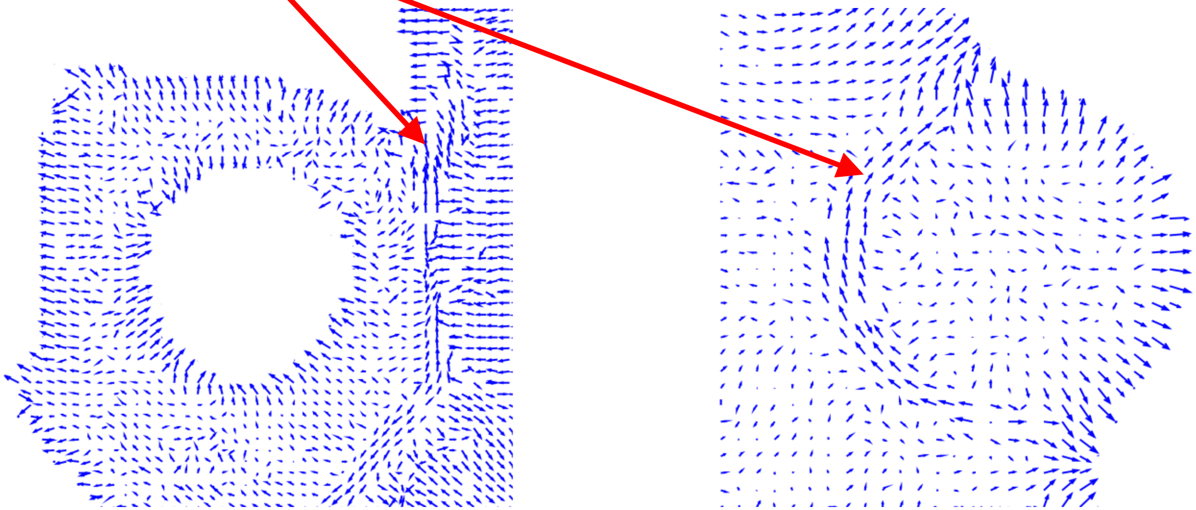
Single View Vision: Accuracy



Single View Vision: Limitations

- Ambiguities
- Roughness modifies dependance on zenith angle.
- At specularities use wrong $\rho(\theta)$
- Inter-reflections

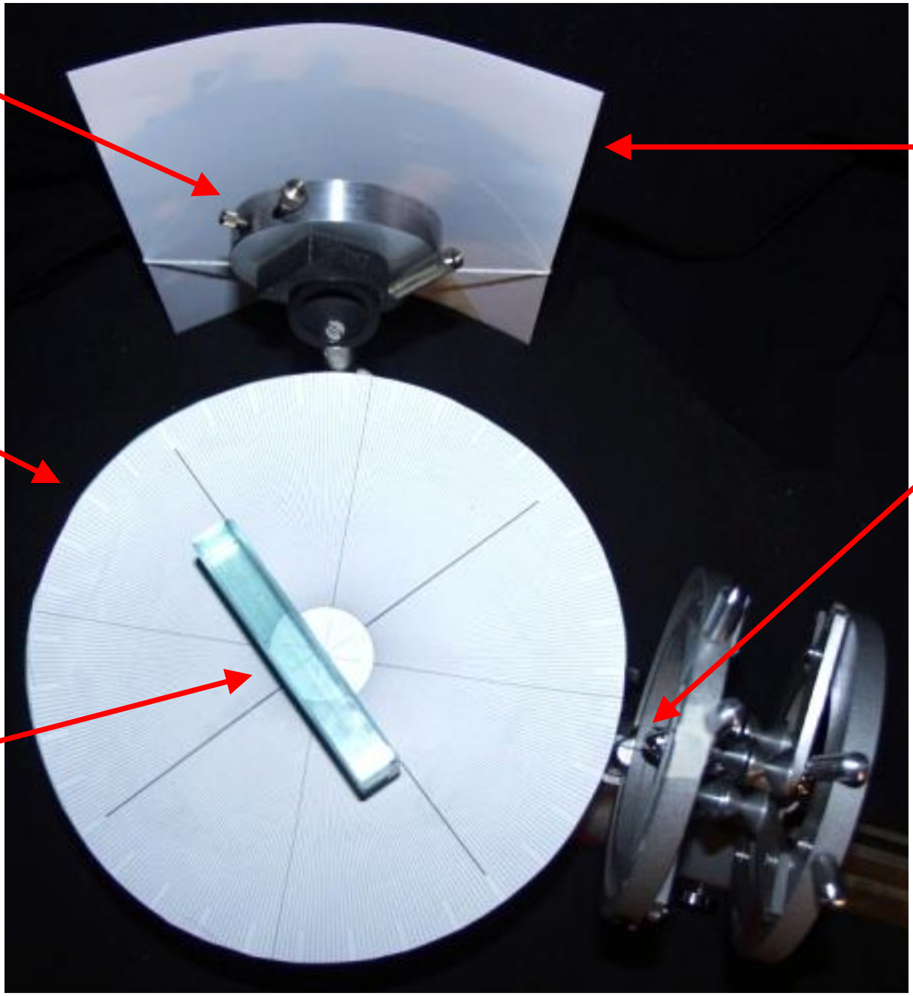




Single View Vision: Refractive Index

Linear polarizer
Placed horizontally

Screen



Rotating table

Laser

Sample

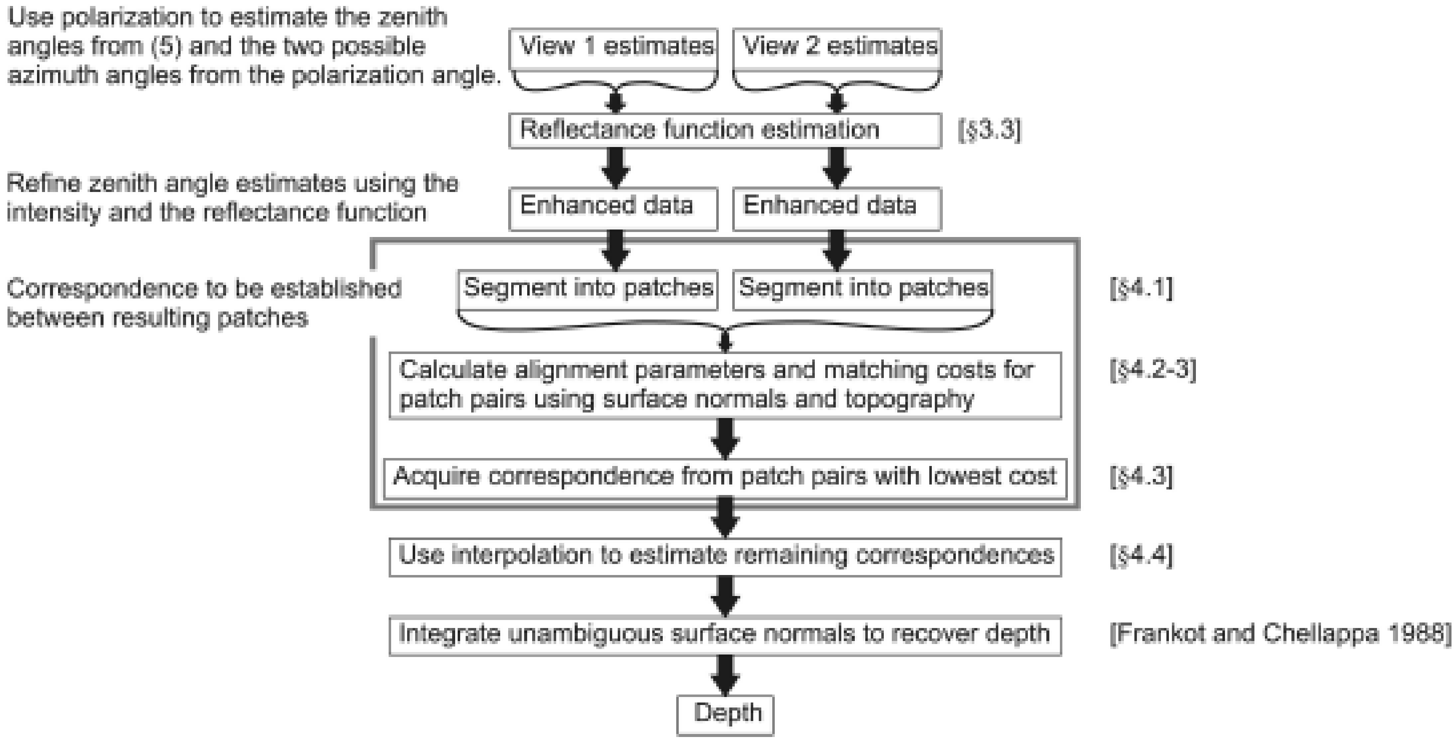
Resolving some of the problems: PAMI07

- Make radiance function estimation to recover azimuth angle more accurately (implemented as look-up table).

- Use multi-view correspondences to resolve ambiguities in surface normal direction.

- Increase object coverage using multiple views.

Schematic for multi-view approach



BRDF Estimation

Use estimates of zenith and azimuth angles to explore angular dependence of reflected surface radiance.

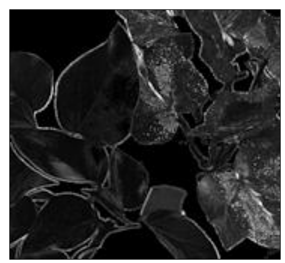
Polarisation Image

- Light through a polaroid: $I(\alpha_p) = I_\circ \left[1 + \rho \cos(2\alpha_p - 2\phi)\right]$ ...(1)

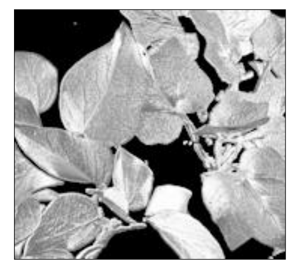
- Degree of diffuse polarisation

$$\rho = \frac{(n - 1/n)^2 \sin^2\theta}{2 + 2n^2 - (n - 1/n)^2 \sin^2\theta + 4\cos\theta\sqrt{n^2 - \sin^2\theta}} \qquad \text{…(2)}$$

Mean Intensity I_o



Polarisation Degree ρ

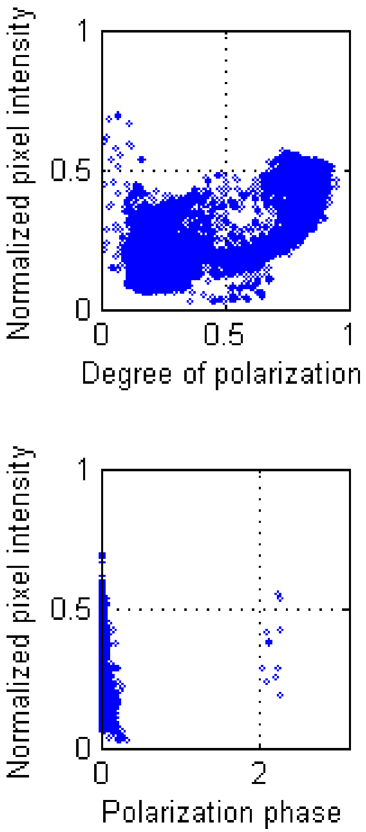


Polarisation Phase ϕ

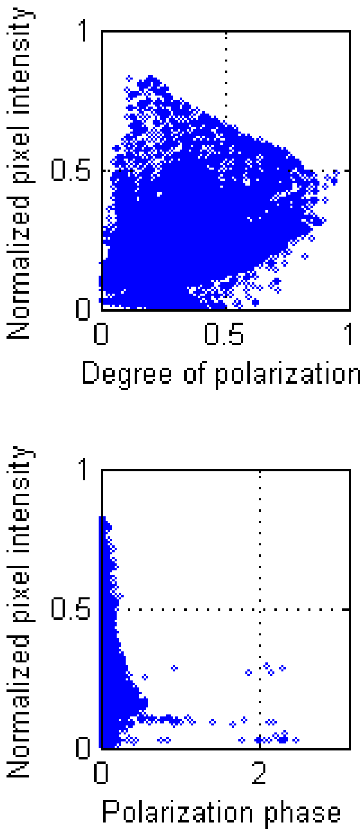
- From the Fresnel theory, azimuth angle equals ϕ in (1) ; zenith angle can be computed from (2) given ρ and n.

Reflectance Distributions

Natural leaves



Plastic leaves



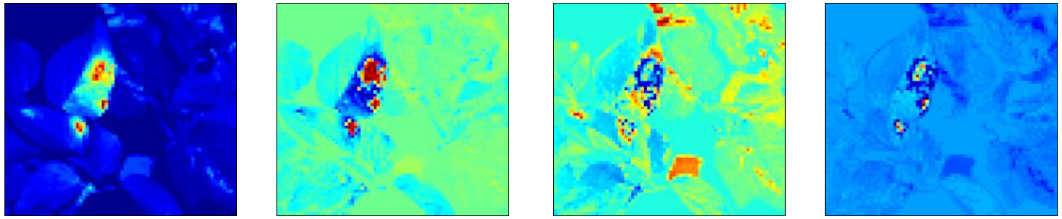
Feature Generation

- Spherical Harmonic coefficient definition

$$a_{l,m} = \int_0^{2\pi}\int_0^{\pi} f(\theta,\phi) Y_l^m(\theta,\phi)\sin\theta \, d\theta \, d\phi \qquad \ldots(3)$$

- Moment estimates of coefficients

$$a_{l,m} = \frac{1}{M}\sum_{i=1}^{M} \hat{I}_i Y_l^m(\theta_i,\phi_i) \qquad \ldots(4)$$



PCA mapped features - 4 dimensions with greatest variance

- Mahalanobis distance between the feature vectors for segmentation by normalized graph cuts.

Experimental Results

- Segmentation Results:

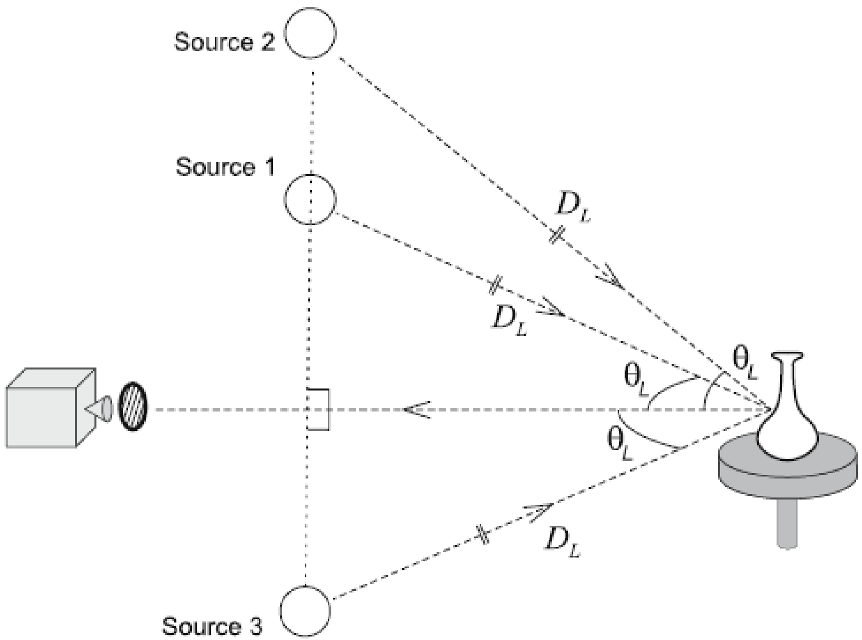
Test Image | *Natural surface* | *Artificial surface*

Photometric stereo

Fixed camera and object, variable light source direction.

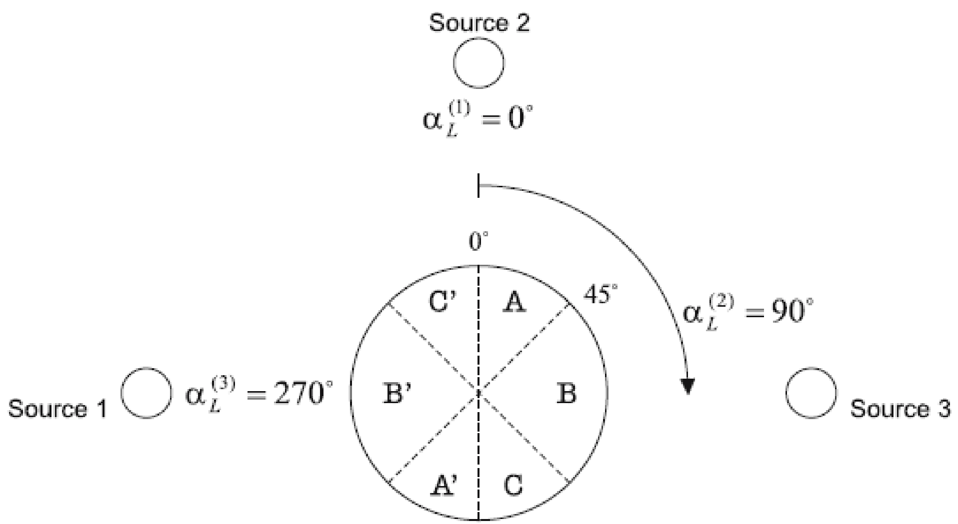
- However, there is an ambiguity of 180 degrees in azimuth angle determination.
- Resolve ambiguity using light source consistency constraints.

- Photometric stereo setup: polarization camera and object kept fixed but light source is moved.



Resolving angle ambiguities

- Azimuth angles are disambiguated differently depending on whether the phase angle is less than 45◦(regions A and A'), between 45◦ and 135◦ (regions B and B') or greater than 135◦(regions C and C').



Light source consistency constraints

- Resolve azimuth angle ambiguities using light source consistency constraints (for spherical surface):

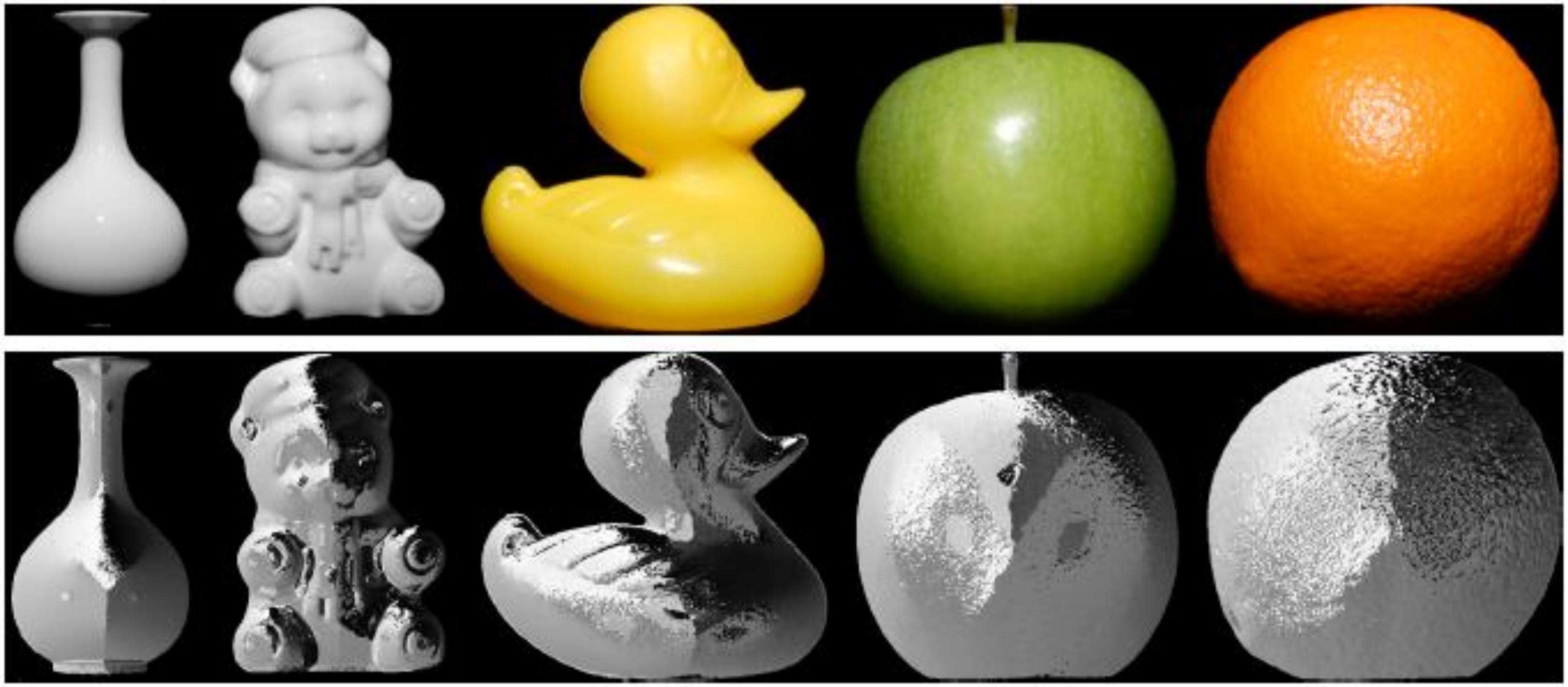
$$\text{if} \quad \phi_k < 45^\circ \quad \text{then} \quad \alpha_k = \begin{cases} \phi_k & \text{if} \quad I_k^{(2)} > I_k^{(1)} \\ \phi_k + 180^\circ & \text{otherwise} \end{cases}$$

$$\text{if} \quad 45^\circ \leq \phi_k < 135^\circ \quad \text{then} \quad \alpha_k = \begin{cases} \phi_k & \text{if} \quad I_k^{(3)} > I_k^{(1)} \\ \phi_k + 180^\circ & \text{otherwise} \end{cases}$$

$$\text{if} \quad 135^\circ \leq \phi_k \quad \text{then} \quad \alpha_k = \begin{cases} \phi_k & \text{if} \quad I_k^{(3)} > I_k^{(2)} \\ \phi_k + 180^\circ & \text{otherwise} \end{cases}$$

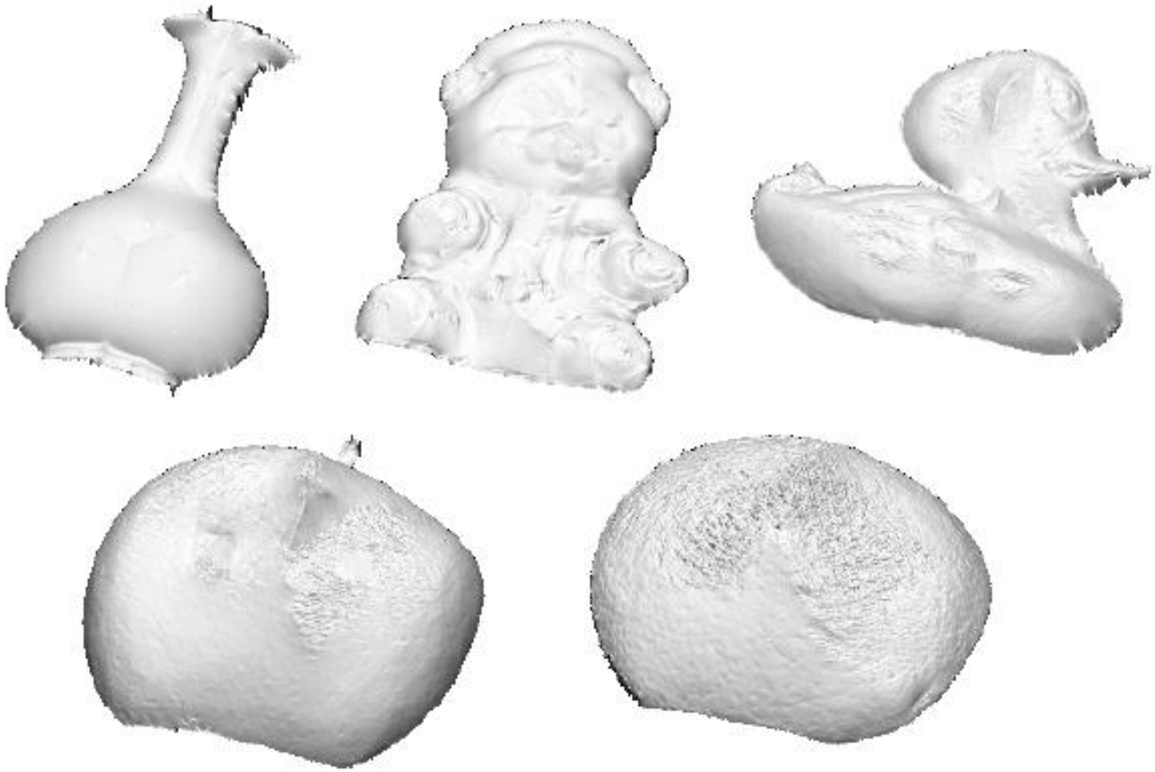
Examples of disambiguation

- Raw images (top) and disambiguated azimuthal angles (bottom):



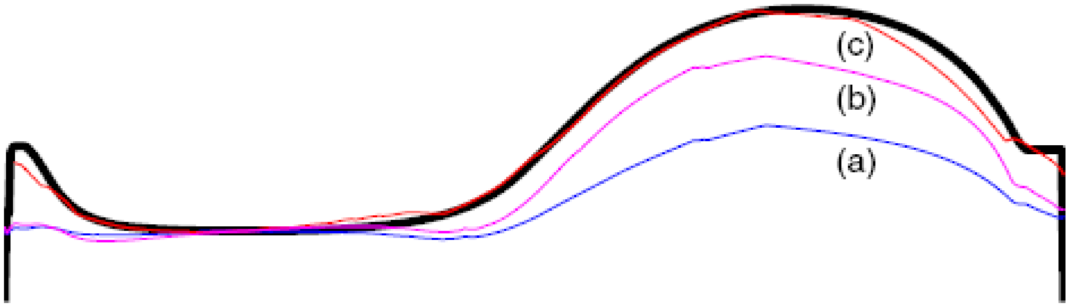
Results

- Reconstructed surfaces



Accuracy

- Comparing with ground truth range data:



Profile of the vase reconstruction using

(a) raw zenith angle estimates,

(b) zenith angles estimated using the Miyazaki et al. method,

(c) zenith angles using method reported here,

(d) exact profile is shown by the thick line.

Shape and Refractive iIndex from Spectro-polarimetry

Idea

- Multiple polarisation images from a single viewpoint and different wavelengths

- Additional constraints on a) wavelength and b) surface integrability.

- Solved using optmisation method,

Physics

From the Fresnel equations

$$\frac{I_{min}}{I_{max}} = \left(\frac{\cos\theta(u)\sqrt{\eta^2(u,\lambda) - \sin^2\theta(u)} + \sin^2\theta(u)}{\eta(u,\lambda)} \right)^2 \quad = R(u,\lambda)^2$$

Solve for zenith angle

$$\sin\theta(u) \equiv \frac{\eta(u,\lambda)\sqrt{1 - R^2(u,\lambda)}}{\sqrt{\eta^2(u,\lambda) - 2R(u,\lambda)\eta(u,\lambda) + 1}}.$$

Material Dispersion Equations

Need model of wavelength dependence of refractive index

Cauchy

$$\eta(u,\lambda) = \sum_{m=1}^{M} C_m(u)\lambda^{-2(m-1)},$$

Sellmeier

$$\eta^2(u,\lambda) = 1 + \sum_{m=1}^{M} \frac{B_m(u)\lambda^2}{\lambda^2 - D_m(u)},$$

Integrability Constraint

Constraints on second mixed derivatives of height function can be rexpressed in terms of variation in zenith and azimuth angles.

$$\cos\alpha(u)\frac{\partial\tan\theta(u)}{\partial y}=\sin\alpha(u)\frac{\partial\tan\theta(u)}{\partial x}.$$

In terms of rate of change of zenith angle with position on pixel lattice

$$\cos\alpha(u)\theta_y(u)=\sin\alpha(u)\theta_x(u)$$

Cost Function

Sum of terms from data-closeness of Fresnel transmission ratio and integrability constraint over image pixels and wvalengths

$$\mathcal{E} = \int_{\mathcal{S}} \int_{\mathcal{W}} \left(R(u,\lambda) - r(u,\lambda) \right)^2 d\lambda \, du$$
$$+ \beta(u) \int_{\mathcal{S}} \left(\cos\alpha(u)\theta_y(u) - \sin\alpha(u)\theta_x(u) \right)^2 du,$$

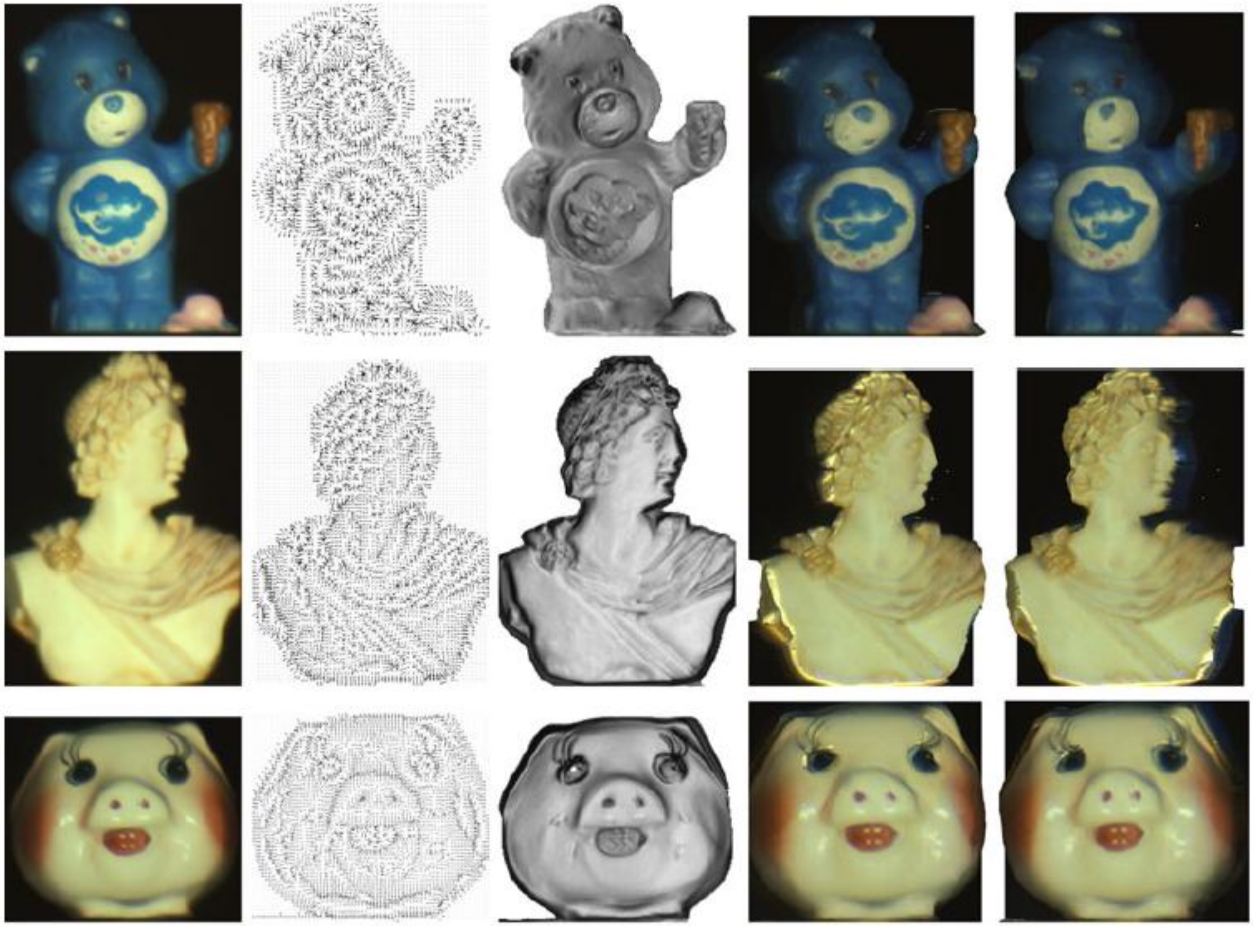
Method

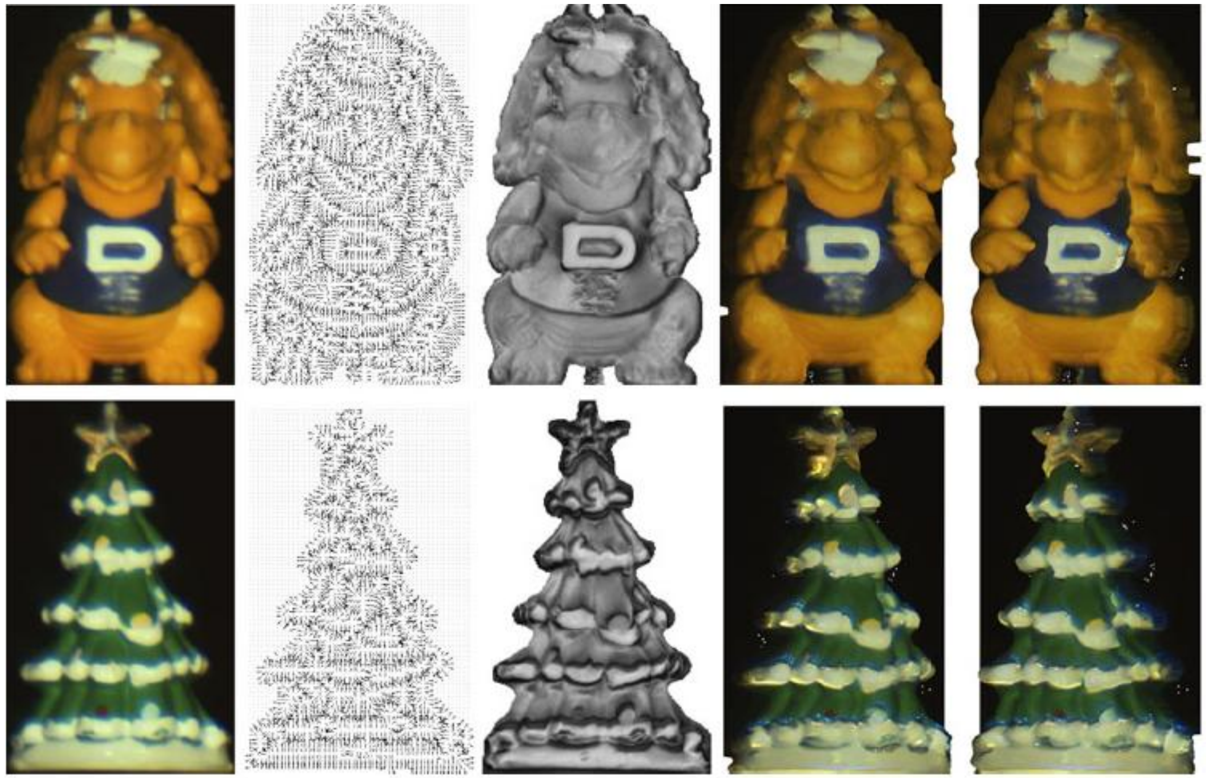
- Collect images with fixed viewpoints at different light source, polariser and wavelength settings.

- Solve minimisation problems for refractive index and zenith angle

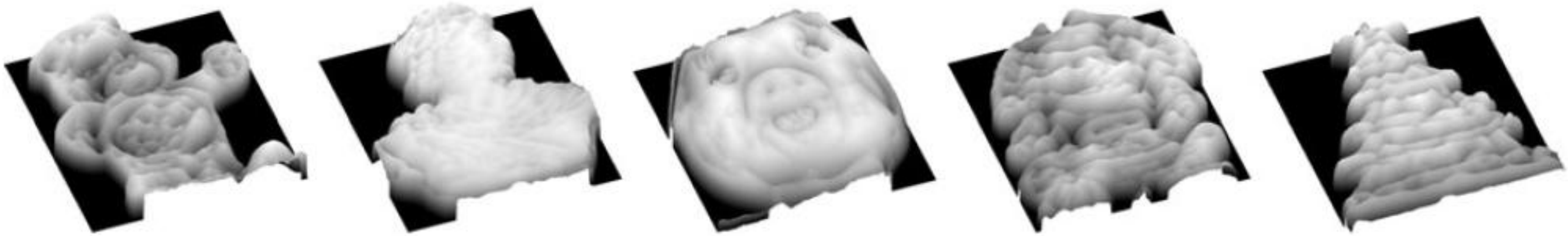
- Use wavelength dependant phase information for resolve azimuth anlge ambiguity.

Results

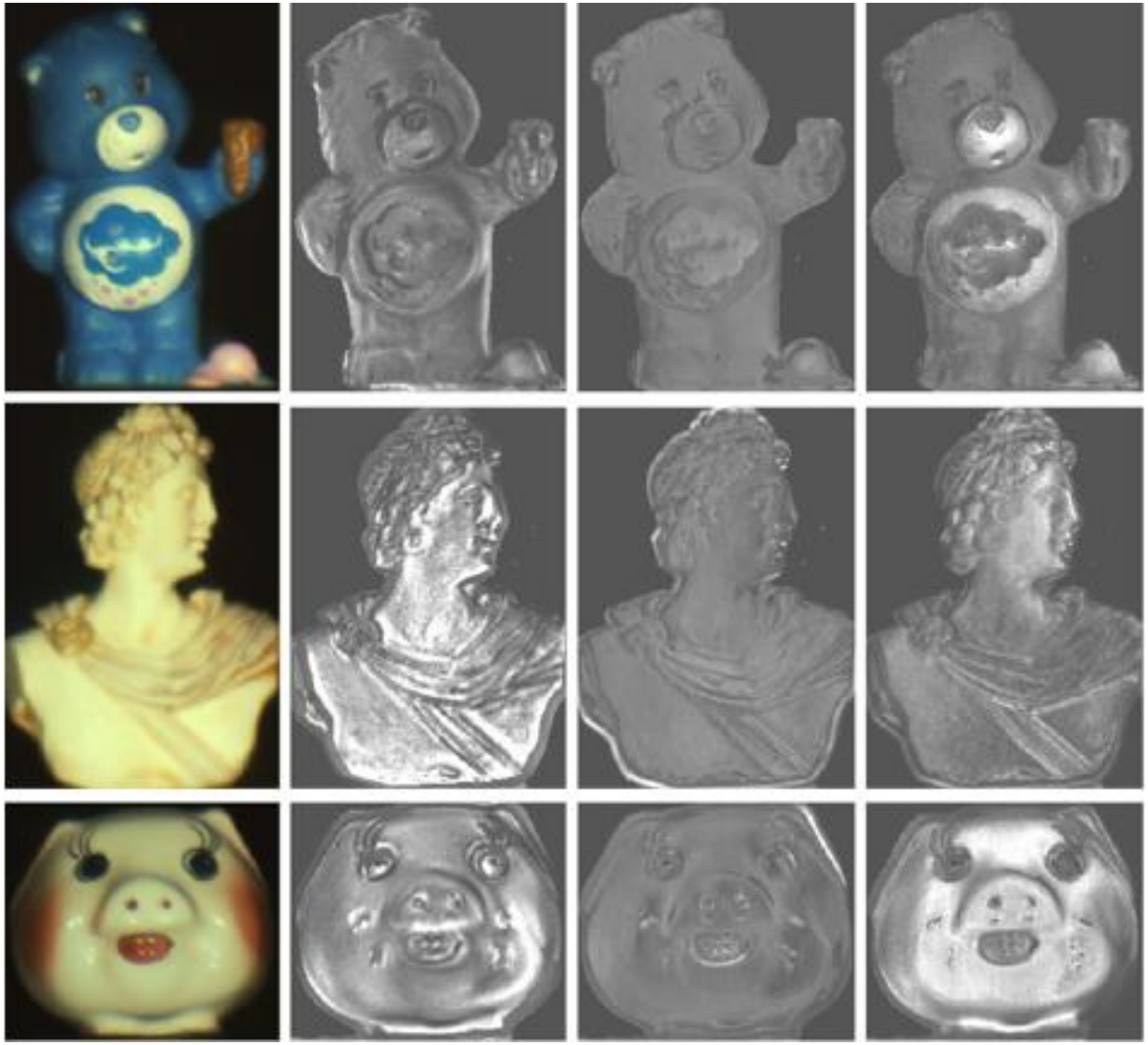




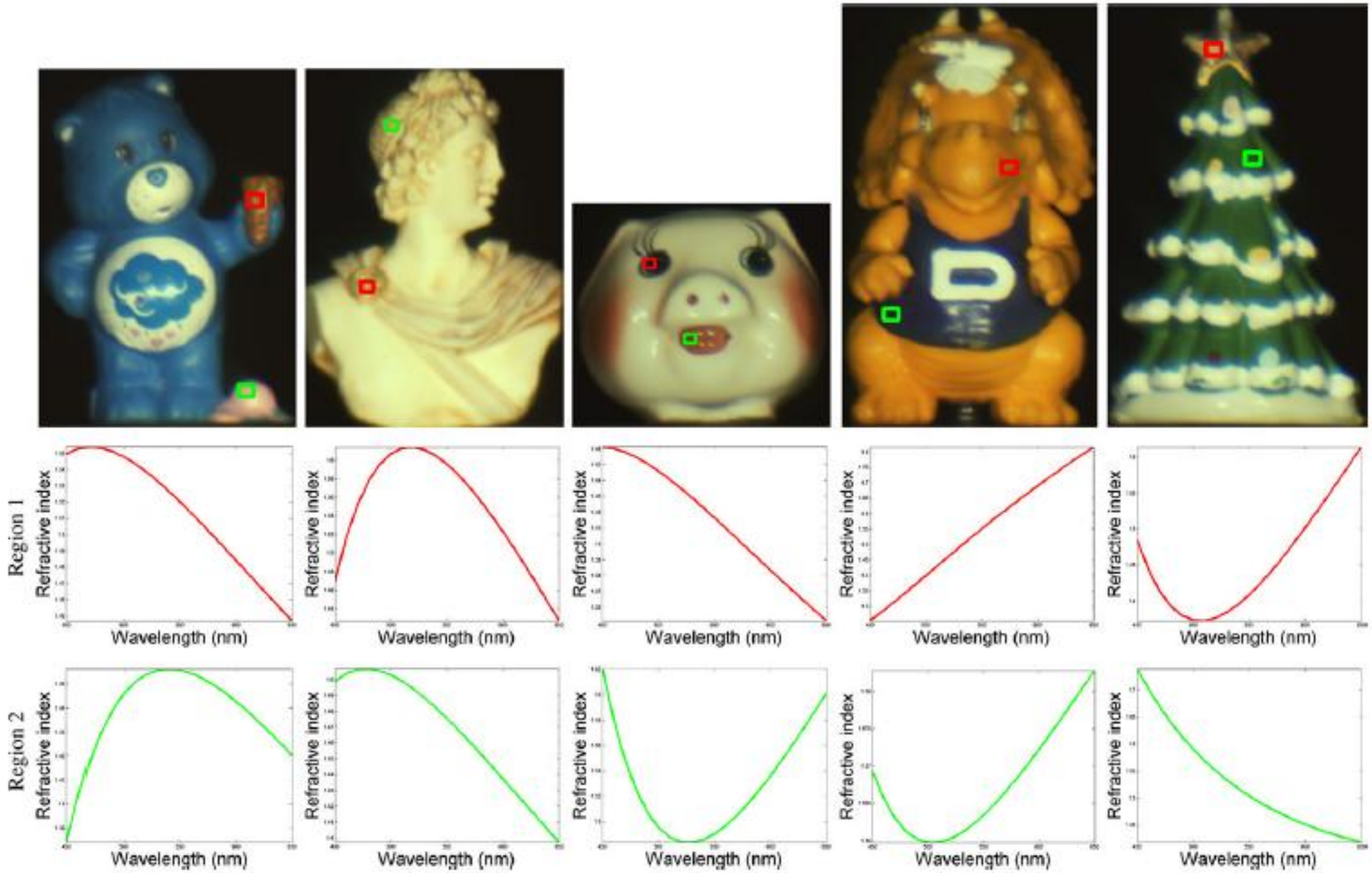
Depth maps



Refractive Index Variation



Wavelength Dependence of Refractive Index



Conclusions

- Demonstrated potential of diffuse polarisation for shape-recovery from single and multiple polarisation images.

- Gives reliable shape recovery, and could be the basis of a range imaging camera design.

- Currently exploring posibilities of using method for fruit quality control.